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A dominance solvable global game with strategic substitutes☆

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ABSTRACT

Global games emerged as an approach to equilibrium selection. For a general setting with supermodular payoffs, unique selection of equilibrium has been obtained through iterative elimination of strictly dominated strategies. For the case of global games with strategic substitutes, uniqueness of equilibrium has not been proved by iterative elimination of strictly dominated strategies, making the equilibrium less appealing. In this work we provide a condition for dominance solvability in a simple three-player binary-action global game with strategic substitutes. This opens an unexplored research agenda on the study of global games with strategic substitutes.

1. Introduction

Global games are games of incomplete information, where the players' payoffs depend on an uncertain state that represents the fundamental of the modeled situation, from which each player receives a private signal with a small amount of noise. In these games, the noise technology is common knowledge, so each player's signal generates beliefs about fundamentals of the model and the other players' beliefs (over fundamentals and beliefs of their rivals and so on). Incomplete information comes from a noisy payoff perturbation of a complete information game in a way that when the noise vanishes we recover the original game. Originally, global games were assessed as equilibrium selection devices and, in time, they have become as well a useful methodology to simplify the analysis of high-order beliefs in strategic settings. Our interest relates to their equilibrium selection application.

Global games were first introduced by Carlsson and van Damme (1993) as a means to depart from the assumption that players are excessively rational and well-informed with respect to the real-life situation under scrutiny. The idea behind this equilibrium selection approach is to examine the set of Nash equilibria of a game as a limit of equilibria of payoff-perturbed games and observe any reduction in the set. For a given realization of the state and its associated complete information game, the global game approach may allow selection of a unique equilibrium in this game, provided that there is a unique equilibrium in the incomplete information game that results when the noise in the players' observation is sufficiently small.

Carlsson and van Damme (1993) show that for a general class of two-player, two-action games, this limit comprises a single equilibrium profile. Moreover, the equilibrium profile is obtained through iterative elimination of strictly dominated strategies (henceforth IESDS). Roughly, the deletion requires that, for each player and for each action of that player, there are certain extreme values of the state, for which that action is strictly dominant. Even if these values carry very little probability weight, the players can use signals close to these "dominance regions" to rule out certain types of behavior of others. Hence, the iterative deletion proceeds. These results have been extended by Frankel et al. (2003) to a more general class of global games with strategic complementarities which have been useful for the study of economic models such as bank runs (Goldstein and Pauzner, 2004), currency crises (Morris and Shin, 1998) and herding behavior (Chamley, 1999), among others.[1]

Most of the global game literature has been developed in the context of strategic complementarities. Recent results, mainly re-

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[1] For a survey of the global games literature see Morris and Shin (2003).

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lated to the question of noise-independent selection of equilibrium, are presented in Basteck et al. (2013), Oury (2013), Honda (2011), Oyama and Takahashi (2011) and Basteck and Daniëls (2011). Noise-independence selection of equilibrium is obtained when the selected equilibrium does not depend on the choice of the noise distribution.

Global games with strategic substitutes have not been as thoroughly studied as the case of strategic complements. Uniqueness of equilibrium cannot be obtained by simply passing from the strategic complements model of Frankel et al. (2003) to the strategic substitutes environment. The elimination of strictly dominated strategies may not provide a unique outcome and so this technique cannot be used to prove uniqueness of equilibrium. However, by adding a minimum of player heterogeneity, Harrison (2003) showed that the equilibrium is unique in a fairly general model with strategic substitutes. Still, this unique equilibrium may not be the unique outcome of the IESDS (Morris, 2009). In accord with the literature on *strong rationality* the predictive power of the global game approach for equilibrium selection comes not only from uniqueness of equilibrium but also from the method by which this equilibrium is obtained. This is one of the reasons why we are interested in dominance solvability.[2]

In the light of dominance solvability results in games with strategic substitutes and complete information (Zimper, 2007; Guesnerie and Jara-Moroni, 2011), further requirements should allow stating that this unique equilibrium is in fact the only strategy profile that survives IESDS. In this article we study a simple three-player global game with strategic substitutes with heterogeneous players that satisfies the conditions for uniqueness of equilibrium of the theorem in Harrison (2003).[3] We show that if players are sufficiently heterogeneous, the process of IESDS delivers the unique equilibrium profile. Moreover, the selected equilibrium does not depend on the choice of noise structure. This result solves a puzzle in the global games literature and resembles the results found in Zimper (2007) and Guesnerie and Jara-Moroni (2011), regarding the passage from strategic complements to strategic substitutes. It is indeed possible to obtain dominance solvability under strategic substitutes, but in Harrison (2003) and Morris (2009) we see that uniqueness of equilibrium is not sufficient as in the case of strategic complements and thus additional conditions must be required.

2. A simple global game with strategic substitutes

In this section we introduce the global game under scrutiny. We present a binary-action three-player game with strategic substitutes and heterogeneous players.

2.1. Setting

Consider a three-player binary-action game characterized by the payoffs $u_i : \{0, 1\}^3 \times \mathbb{R} \rightarrow \mathbb{R}$,[4]

$$u_i(a_i, a_{-i}, x) := a_i \left(\frac{d}{2} (3 - a_1 - a_2 - a_3) + mx - c_i \right)$$

for $i \in \{1, 2, 3\}$

where $m > 0$, $d > 0$ represents the degree of strategic substitution[5] and c_i may be interpreted as player i's specific cost. Heterogeneity of players is introduced by assuming that $0 < c_1 < c_2 < c_3$.

Note that player i's payoff function is of the form $u_i(a_i, a_{-i}, x) = \pi_i\left(a_i, \sum_{j \neq i} a_j, x\right)$, where $\pi_i : \{0, 1\} \times \{0, 1, 2\} \times \mathbb{R} \rightarrow \mathbb{R}$ is an auxiliary function defined by

$$\pi_i(a_i, n, x) := a_i \left(\frac{d}{2} (3 - a_i - n) + mx - c_i \right)$$

that depends on other players' actions through their sum (the number of players – other than i – that are choosing action 1).

Let us define $\Delta\pi_i(n, x) = \pi_i(1, n, x) - \pi_i(0, n, x)$ as the net gain of player i of playing 1 instead of 0. Then

$$\Delta\pi_i(n, x) = \Delta\pi(n, x) - c_i$$

where

$$\Delta\pi(n, x) := \frac{d}{2} (2 - n) + mx. \tag{1}$$

Note that in this model

$$\Delta\pi_i(n, x) - \Delta\pi_i(n + 1, x) = \tfrac{d}{2} \tag{2}$$

and since $d > 0$, the incentive to choose the higher action is decreasing in the actions of the rivals, so the game indeed presents strategic substitutes.[6] The greater the value of d, the steeper the incentive to play the higher action.

Finally, let us define $k_i(n)$ by the unique solution in x of

$$\Delta\pi_i(n, x) = 0 \quad \Longrightarrow k_i(n) = \frac{c_i - \frac{d}{2} (2 - n)}{m}.$$

The value $k_i(n)$ allows us to identify the optimal action of player i when n of her opponents are playing 1. If $x < k_i(n)$ then player i will choose action 0 and if $x > k_i(n)$ then player i will choose action 1. Note that $k_i(n)$ is increasing in n and that $k_1(n) < k_2(n) < k_3(n)$ for all n. We will denote

$$\underline{k}_i = k_i(0) \qquad k_i = k_i(1) \qquad \bar{k}_i = k_i(2)\,.$$

We see then that if $x > \bar{k}_i$ then player i will optimally choose action 1 regardless of the number of opponents playing 1. Equivalently, if $x < \underline{k}_i$ then player i will optimally choose action 0 regardless of the number of opponents playing 1. In the global game literature, these intervals are called upper and lower dominance regions, respectively.

If there is complete information, depending on the value of x the game may have: multiple equilibria, a unique equilibrium, a unique equilibrium with two players playing strictly dominant strategies or a unique equilibrium in strictly dominant strategies (when x is in the dominance regions of all the players). Fig. 1 depicts the type of equilibria depending on the value of x and the dominance regions for each player.[7]

Since we have multiplicity of equilibria for some values of x, we are interested in using the global game approach for equilibrium selection. However, we are not only interested in uniqueness of equilibrium under incomplete information but also in the possibility that this profile is obtained through IESDS.

2.2. Incomplete information

Consider now the three-player incomplete information game $\Gamma(\sigma)$, consisting of the previous payoff structure and where each player has some uncertainty about x. Instead of observing the

[2] The concept of strongly rational equilibrium was first stated by Guesnerie (1992) as a means to provide an eductive foundation for the rational expectations hypothesis. An equilibrium is strongly rational, if it is the only *rationalizable strategy profile* of a game (Guesnerie, 1992, 2002). Dominance solvability implies strong rationality of the equilibrium.

[3] The simplest unexplored case is the three-player binary-action game, since results in two-player global games may be derived from Carlsson and van Damme (1993).

[4] This game is inspired by the game presented in Morris (2009).

[5] See (2).

[6] A negative d would model a game with strategic complements.

[7] For illustrative purposes, in Fig. 1 we have placed the values $k_i(n)$ such that $\underline{k}_3 < k_1$ and $k_3 < \bar{k}_1$, but this is not an assumption in what follows.