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Non-anonymous ballot aggregation: An axiomatic generalization of Approval Voting

Jorge Alcalde-Unzu [a,*], Marc Vorsatz [b,c]

[a] Department of Economics, Public University of Navarre, Campus Arrosadia, 31006 Pamplona, Spain

[b] Departamento de Análisis Económico II, Universidad Nacional de Educación a Distancia (UNED), Paseo Senda del Rey 11, 28040 Madrid, Spain

[c] Fundación de Estudios de Economía Aplicada (FEDEA), Calle Jorge Juan 46, 28001 Madrid, Spain

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ABSTRACT

We study axiomatically situations in which the society agrees to treat voters with different characteristics distinctly. In this setting, we propose a set of intuitive axioms and show that they jointly characterize a new class of voting procedures, called Type-weighted Approval Voting. According to this family, each voter has a strictly positive and finite weight (the weight is necessarily the same for all voters with the same characteristics) and the alternative with the highest number of weighted votes is elected. The implemented voting procedure reduces to Approval Voting in case all voters are identical or the procedure assigns the same weight to all types. Using this idea, we also obtain a new characterization of Approval Voting.

1. Introduction

Motivation. There are many instances in which the members of a society or an institution vote in order to take a decision and each voter's impact on the outcome depends on her/his underlying characteristics. Examples include the EU Member Council or the IMF Board of Directors, where the weight of a country is determined by its population size or its stake, respectively (see, Tables 1 and 2 in the Appendix); management boards, where the vote of the CEO tends to count double in case of a tie; or hiring decisions in academic institutions, where the opinion of senior members is usually given more weight. From a theoretical point of view, this implies that voters are not treated equally and that existing axiomatic results on the question of which voting procedure to implement do not directly apply. It is consequently the aim of this study to complement the existing literature on axiomatic voting theory by suggesting a general class of voting procedures that is able to cover these kinds of situations.

The aggregation procedures discussed in the literature differ essentially in the type of information they take into account from the individual preferences. For example, *Plurality Voting*, the most widely used voting procedure, allows each individual to indicate only her most preferred alternative (and the alternative with most votes is elected). One common critique of Plurality Voting is that it may actually result in the election of the worst alternative for a majority of individuals even in single-winner elections. As a simple example, consider the case when there are three alternatives, two of which are very similar. Then, if the votes for the two similar alternatives are distributed equally, the third alternative may be elected even though a majority of the voters would prefer either of the other two alternatives.

Approval Voting, introduced by Brams and Fishburn (1978), has been explicitly designed to overcome this drawback of Plurality Voting by allowing individuals to vote for (or approve of) as many alternatives as they wish to. As usual, the alternative with most votes wins the election. Recent evidence from field experiments by Laslier and Van der Straeten (2008) in France and Alós-Ferrer and Granić (2012) in Germany has shown that Approval Voting modifies the overall ranking of the alternatives and that it tends to elect the alternative that is most widely accepted in the population. This is the main reason why we deviate from using Plurality Voting as a benchmark and frame our analysis in the (more general and more complex) context when individuals can approve any number of alternatives.

Characterizations. We are interested in general voting procedures that are operable in different voting environments in which the set of voters and the set of alternatives might vary. In particular, given a population of potential voters and a conceivable set of alternatives, a voting procedure should specify an outcome (a non-empty subset of the set of feasible alternatives) for every electorate (the individuals that indeed vote) and every set of feasible

* Corresponding author. Tel.: +34 948169350; fax: +34 948169721.
E-mail addresses: jorge.alcalde@unavarra.es (J. Alcalde-Unzu), mvorsatz@cee.uned.es (M. Vorsatz).

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alternatives (the alternatives actually standing for election). We also assume that voters are partitioned into types according to some exogenous characteristics in such a way that individuals with equally relevant characteristics belong to the same type. In the examples of indirect democracy mentioned earlier, one can think of classifying voters into types in function of the number of people or the stake the voter represents. In problems of decision making in small groups, the voter's type could be associated with some of her personal characteristics such as seniority, age, *etc.*

In this setting, we consider a set of intuitive properties. First, we introduce two consistency properties that impose some structure on how the result of the voting procedure should be adapted when the set of alternatives or the set of individuals change: *Consistency in alternatives*, which is the analogue of Arrow's Choice Axiom, states that if the set of feasible alternatives is reduced yet some of the originally elected alternatives remain feasible, then exactly those alternatives have to be elected in the new situation; and *Consistency in voters*, which requires that if two disjoint electorates select a common set out of two feasible alternatives, then exactly this set has to be elected when the two electorates are assembled. Afterwards, we consider two symmetry properties: *Symmetry across types*, which means that voters of the same type have to be treated equally; and *Symmetry across alternatives*, which is the classical neutrality property. Finally, we add two well-known conditions: *Faithfulness*, which asks that if there is a single voter who approves x but not y, then x has to be elected whenever x and y are the only two feasible alternatives; and *Continuity* which, roughly speaking, states that no group of individuals should be able to always impose completely its opinion on the result of an election when joined with a sufficiently large electorate formed by many subgroups that agree among them on the set of alternatives that has to be selected.

Our first result, Theorem 1, shows that these properties fully characterize a general class of voting procedures that we will call *Type-weighted Approval Voting*. Each voting procedure of this family is associated with a vector of strictly positive and finite weights, one for each type of voter, and the winning alternative is the one with the highest number of weighted votes. If no discrimination across types have sense in a particular context, all weights should be equal and the voting procedure reduces to Approval Voting. Exploiting this fact, we show in our second result, Theorem 2, that if Symmetry across types is strengthened to the classical condition of *Anonymity* (Symmetry across voters), one essentially obtains a new characterization of Approval Voting in which Faithfulness and Continuity are eliminated as necessary requirements.

Related literature. Our work contributes to the existing literature on axiomatic voting theory. Roberts (1991) was the first to characterize Plurality Voting. Richelson (1978), Ching (1996), and Yeh (2008) also characterize the Plurality Rule, but as a social choice correspondence and not as a voting procedure; that is, in these studies, the domain is the Cartesian product of all linear orders on the set of alternatives. Fishburn (1978, 1979), Sertel (1988), Baigent and Xu (1991), Goodin and List (2006), Vorsatz (2007) and Sato (2013) provide different characterizations of Approval Voting. Alós-Ferrer (2006) shows that the properties in one of Fishburn's characterizations are not independent. Maniquet and Mongin (2013) study possible social welfare orderings corresponding to Approval Voting and characterize them by Arrow's conditions when preferences are dichotomous. Finally, Massó and Vorsatz (2008) and Alcalde-Unzu and Vorsatz (2009) introduce classes of voting procedures that generalize Approval Voting in natural ways. In Massó and Vorsatz (2008), the neutrality property is relaxed; in Alcalde-Unzu and Vorsatz (2009), the weight of a vote is a decreasing function in the number of approved alternatives.

One can think of Massó and Vorsatz (2008) and the characterization obtained in Theorem 1 as dual approaches that bear important similarities. Massó and Vorsatz (2008) relax neutrality and, as a result, characterize voting rules that assign different weights to alternatives. In this paper, we weaken the classical anonymity property and, as a consequence, weights are assigned to voters. However, there is still one important asymmetry that naturally occurs in the formal analysis. In Massó and Vorsatz (2008), the relative weight between two alternatives can be easily determined because it is known from the voting rule how many votes one alternative has to receive in order to compensate one vote to the other alternative. Yet, the construction of a weighted representation of a voting rule when anonymity is relaxed is more complicated. This is because adding one voter to an election has the effect that the particular weight of this voter has to be determined endogenously as well, and therefore, does not provide sufficient information of how to determine the relative weights of the other voters. Only the additional requirement that voters are divided into types and that there is an infinite population of potential voters of each type allows us to determine the relative weights.

Our second characterization, Theorem 2, also relates to the literature mentioned before. By working with a variable set of alternatives, contrary to the majority of studies found in the literature, we can naturally impose the property of Consistency in alternatives (which ultimately allows the decision maker to go forth and back between social choice and social welfare functions) in substitution of other properties. The only two other studies along the same line that characterize Approval Voting are Vorsatz (2007) and Sato (2013). The former characterizes Approval Voting in a dichotomous preference setting using strategy-proofness. The latter characterizes Approval Voting independently and simultaneously to this paper by using a very similar set of axioms to that imposed in Theorem 2 (see the detailed discussion in Section 3).

2. Notation and definitions

We consider a setting with variable sets of voters and alternatives. Formally, let X be a finite set of *conceivable alternatives*. Generic alternatives will be denoted by x, y, and z; subsets of X by S and T. The cardinality of X, $|X|$, is greater than or equal to 3.[1] The set of *feasible alternatives* K, the alternatives that are actually standing for an election, is a non-empty subset of X. Our analysis focuses on the idea that the individuals participating in the election may differ in their characteristics. To model this, we assume that there is a finite set of *types* $\Theta = \{1, 2, \ldots, \theta\}$ and that for each type $t \in \Theta$, there is an infinite number of *potential voters* I_t. Hence, $I \equiv \bigcup_{t\in\Theta} I_t$ is the *population* of all potential voters. The individuals actually participating in an election, an *electorate* N, is a non-empty and finite subset of the population I. We will also make frequent use of the capital letters A and B to denote electorates.

Each individual $i \in I$ partitions the set of alternatives X into two sets: $M_i \in 2^X$ and $(X \setminus M_i)$. The interpretation is that M_i is the set of alternatives i votes for (or approves of). Thus, we can describe the opinion of an individual i by only referring to the set M_i. A *profile* $M = (M_i)_{i\in I} \in (2^X)^I$ is a list of all votes. Given a profile M and an electorate N, a *response profile* $M_N = (M_i)_{i\in N} \in (2^X)^N$ is the n-tuple of votes coming from the electorate N at profile M. Given the response profile M_N, the number of votes x receives from the individuals of type t who belong to the electorate N is denoted by

[1] If there are only two conceivable alternatives, all results of the paper hold true. The unique difference is that, when $|X| = 2$, one of the axioms, Consistency in alternatives, is superfluous. This will become evident from the proofs.