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Dynamic price competition in aftermarkets with network effects

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ABSTRACT

This paper studies the dynamic price competition between two firms that sell horizontally differentiated durable goods and, subsequently, provide exclusive complementary goods and services to their customers. The paper analyzes how optimal pricing strategies are affected by the existence of network effects associated with the size of firms' consumer base. The interaction is thoroughly analyzed as a continuous time linear–quadratic differential game. We provide a necessary and sufficient condition for the existence of a unique duopoly equilibrium in affine strategies. When this condition holds, we show that optimal pricing strategies crucially depend on the nature of the network effects.

1. Introduction

The value of durable goods often depends on the subsequent consumption of complementary goods and services (CGS) sold in aftermarkets, e.g. smartphones/tablets and applications, cell phone network subscriptions and phone calls, coffee machines and coffee capsules, printers and ink cartridges, cars and maintenance/repairing services…

Shapiro (1995) has identified three key features of aftermarkets: (i) goods/services traded in aftermarkets aim at complementing a durable good/equipment, whose value considerably depends on CGS consumption; (ii) equipment choices precede consumers' purchases of CGS; and (iii) consumers are significantly "locked-in" to their equipment, meaning that the switch from one equipment brand to another is highly unlikely.

We investigate the dynamics of firms' strategic interaction in primary markets and aftermarkets when these markets are subject to network effects.[1] The paper addresses both *primary market network effects* (PMNE) and *aftermarket network effects* (AMNE).

The first occurs when the value of each durable good is increasing with the size of the consumer base of the corresponding producer. For instance, a consumer with a certain smartphone/tablet that is associated with a certain operating system (OS) may get value from the fact that many others use the same OS as it may have some communication benefits or some brand and reputation effect (conspicuous consumption). This generates a direct network effect in the primary market.

Aftermarket network effects arise when the value of CGS sold in the aftermarket depends on the number of consumers owning a similar variant of the durable good. For instance, consumers with a certain smartphone/tablet model (associated with a certain OS) may benefit from the network of the exclusive application(s) which the OS offers as they can chat, play or share files with more users. When this is the case, firms' network size directly affects the value of CGS, yielding direct AMNE. In other contexts, the role played by the size of firms' consumer base may be only indirect, generating indirect AMNE.[2] In the context of the previous example, indirect AMNE may occur, for example, when the number and the quality of the applications made available to a certain smartphone/tablet (associated with a certain OS) depend on the number of individuals owning such a model.

To analyze firms' strategic interaction in primary markets and aftermarkets with network effects, the paper develops a theoretical model of dynamic price competition between two proprietary networks with an aftermarket. Firms produce horizontally differentiated durable goods (in the primary market) and they also supply CGS to locked-in consumers (in the aftermarket). Consumers are forward-looking. When arriving in the primary market, they

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1 Network effects arise when the value of a certain good depends not only on its intrinsic characteristics but also on the consumption choices made by other consumers in the market (see the seminal papers by Rohlfs, 1974, Katz and Shapiro, 1985, Grilo et al., 2001; or, more recently, Amir and Lazzati, 2011 and Griva and Vettas, 2011).

2 See Katz and Shapiro (1994) for further information on the distinction between direct and indirect network effects.

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choose the durable good that maximizes their expected lifetime utility.

The interaction between firms is analyzed as a continuous time differential game with linear transitions and quadratic payoffs, in addition to affine expectation rules for the forward-looking aspect of the (Markov-perfect) equilibrium under consideration.[3] We provide a necessary and sufficient condition for the existence of a unique Linear Markov Perfect Equilibrium (LMPE) in which both firms have non-negative market shares. When this condition holds, instantaneous equilibrium prices in the primary market depend on the degree of differentiation between the durable goods, the profitability of the corresponding aftermarket and the relative size of firms' networks in the primary market.

Regarding the last aspect, we conclude that equilibrium price trajectories in the primary market may be increasing or decreasing with the size of firms' consumer base in the primary market. In particular, the shape of these trajectories depends on the intensity of PMNE *vis-à-vis* the intensity of AMNE. The latter induce increasing marginal returns to firms' network size. As a result of AMNE, equilibrium pricing strategies would be decreasing with the size of firms' consumer base. Instead, PMNE simply increase the attractiveness of the equipment with the largest network. In light of this effect, equilibrium pricing strategies would be increasing with the size of firms' consumer base. Considering the two effects together, we obtain that prices are decreasing (increasing) in firms' market shares in the primary market, when the aftermarket (primary market) network effects are dominant.

In the LMPE steady state, we find that firms share the market evenly. The speed of convergence never exceeds the rate of consumers' exit/entry and the LMPE steady state price is always below the Hotelling benchmark price. Concerning equilibrium trajectories, we find that industry profits and new customers' average expected lifetime utility are decreasing with time. The average equipment price is increasing (resp. decreasing) with time when the intensity of AMNE is sufficiently stronger (weaker) than the intensity of PMNE.

This paper relates to two strands of literature. First, it contributes to the recent literature dealing with dynamic price competition in network industries. Following the seminal papers by Rohlfs (1974) and Katz and Shapiro (1985), a considerable number of works have been devoted to understand the static price competition in network industries. More recently, some work has been devoted to understand the dynamic price competition in network industries (see Garcia and Resende, 2011 for a more detailed survey about this literature). This recent literature includes both computational models (see Markovich, 2008, Markovich and Moenius, 2009, Chen et al., 2009 and Cabral, 2011) and more analytical studies (see, for example, Doganoglu, 2003, Laussel et al., 2004, Mitchell and Skrzypacz, 2006, Driskill, 2007 and Cabral, 2011). Our paper also follows an analytical approach.

Doganoglu (2003) and Mitchell and Skrzypacz (2006) study the dynamic price competition in markets with delayed network effects. Differently, we deal with forward-looking agents. Driskill (2007) presents a model with dynamic network externalities, investigating the properties of the equilibrium that arises when there is monopoly or oligopoly supply. He derives the equilibrium output paths under the assumptions of commitment and Markov strategies. Differently, our model concentrates on the price competition between the durable goods.

Laussel et al. (2004) study the dynamic price competition between two horizontally differentiated "profit maximizing clubs" with congestion effects. Focusing on the LMPE of the dynamic game, they find that access prices are always decreasing with the size of clubs' memberships (due to the negative network effects). This is the closest paper to our work. However, Laussel et al. (2004), like most of the papers on price or quantity competition in network industries, focus on consumers' preferences with additively separable network effects. Hence, they are not able to fully capture neither the demand-side increasing returns produced by network effects (as the ones arising in this paper, which are crucial when AMNE are sufficiently stronger than PMNE) nor the viability problem. A remarkable exception is the recent work by Amir and Lazzati (2011) who analyze general demand functions with non-separable network effects in a static setting.

Cabral (2011) also considers the dynamic price competition between two proprietary networks. He proposes a discrete-time model in which (i) there is a finite number of consumers with idiosyncratic preferences, and (ii) consumers' birth and death processes are stochastic, generating stochastic dynamics. This stochastic framework has the advantage over a deterministic one like ours of allowing the durable coexistence of the firms, without additional conditions limiting the strength of the network effects. This model is however too much complicated to get analytical results outside some very special cases.[4] More precisely, the analytical solution of his model was only possible for: the two consumers case (Proposition 1), the zero discount factor case (Proposition 3, derives by extension, results for a discount factor close to zero) and the case in which there are no aftermarket profits (Proposition 5). While Cabral's model is admittedly more general than ours, his "theoretical results apply only to extreme values of key parameters", as he puts himself. By contrast, our approach, a deterministic continuous time model, though less general, provides the analytical results that are valid on a broader range of parameters. In the core of the paper, we shall compare more precisely the results in each paper. This paper is also related to the literature on aftermarkets (see, for example, Borenstein et al., 2000, Shapiro, 1995, Morita and Waldman, 2004, Chen and Ross, 1993 or Chen et al., 1998). We add to this literature by investigating the interplay between the primary markets and aftermarkets when the network effects take place.

The rest of the paper is organized as follows. Section 2 presents the basic ingredients of the model. Section 3 describes the LMPE of the dynamic game. The strategic implications of firms' equilibrium behavior are studied in Section 4. Finally, Section 5 concludes.

2. The model

This section describes a model of dynamic price competition between two firms that sell horizontally differentiated goods (in the primary market) and provide CGS to their base of consumers (in the aftermarket). At each instant of time, there is an inflow of new consumers (arriving at rate μ) and an identical outflow of old consumers. There are network effects associated with the size of firms' consumer base for the durable good. In this respect, the model encompasses the possibility of primary market and/or aftermarket network effects. PMNE take place when the value of each durable good is increasing with the size of the consumer base of the corresponding producer. AMNE occur when the size of firms' consumer base in the primary market subsequently affects the value of CGS. In what follows, we provide a detailed description of the main ingredients of the model.

In the primary market, firms produce infinitely-lived and horizontally differentiated durable goods (equipment 1 and equipment 2). Without any loss of generality, we assume a zero

3 A book treatment account of the theory and applications of dynamic games in continuous time is Dockner et al. (2000). For a broad survey of applications of the discrete time case, see Amir (1996, 2003).

4 In the case of only three consumers, for instance, solving the model means solving a system of ten non-linear equations with ten unknowns.