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The geometry of revealed preference

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ABSTRACT

In this paper, we examine how the geometry underlying revealed preference determines the set of preferences that can be revealed by choices. Specifically, given an arbitrary binary relation defined on a finite set, we ask if and when there exists a data set which can generate the given relation through revealed preference. We show that the dimension of the consumption space affects the set of revealed preference relations. If the consumption space has more goods than observations, any revealed preference relation can arise. Conversely, if the consumption space has low dimension relative to the number of observations, then there exist both rational and irrational preference relations that can never be revealed by choices.

1. Introduction

Classical revealed preference theory provides a simple, intuitive and nonparametric way of testing the most basic assumption of economics—that agents are rational. In this approach, observed choices by individuals "reveal" a (potentially incomplete) preference relation over the set of consumption bundles. It is well known that the necessary and sufficient condition for the observed choices of an individual to be consistent with utility maximization is that the preference relation revealed by the choices should be acyclic or equivalently should satisfy the Generalized Axiom of Revealed Preference or GARP (Varian, 1982). There is a large body of empirical work that checks this condition in a variety of different settings both in the field and in the lab (references can be found in the survey by Varian, 2007).

Revealed preference in the standard consumption setting is a geometric property—a chosen bundle is revealed preferred to all the bundles that were affordable but not chosen. Put differently, revealed preference is determined by where points corresponding to choices lie in the consumption space relative to the planes determined by the budgets. A bundle is revealed preferred to another if the latter lies underneath the budget plane on which the former lies. In this paper, we examine how this geometry underlying revealed preference determines the set of possible preference relations that can be revealed by choices. This provides ex-ante information about the preference relations that are possible given the number of goods and the number of observations in the data and this can be useful for experimental design. Formally, suppose we are given a set $\{1, \ldots, N\}$ with a relation $\succ$ defined on it. We ask whether there exists a price consumption data set $\{p_i, x_i\}_{i=1}^N$ consisting of K goods such that for all $i \neq j$, $p_i x_i > p_i x_j$ whenever $i \succ j$ and $p_i x_i < p_i x_j$ whenever $i \not\succ j$. The analysis of the above question involves examining whether budget sets can be chosen to intersect in appropriate ways to allow for choices of consumption bundles which will generate relation $\succ$ through revealed preference.

All possible relations may not arise from revealed preference as it may not be possible to separate the consumption space into the required number of regions using "downward sloping" budget planes. Hence, the set of possible revealed preference relations may depend on the dimension K of the consumption space. A higher dimensional consumption space may allow for the budget planes to separate more regions in the space potentially leading to a larger set of revealed preference relations. The main aim of this paper is to examine the relationship between the number of goods in a data set and the set of relations that can be generated through revealed preference. It is well known that when there are only two goods ($K = 2$), the Weak Axiom of Revealed Preference (WARP) is equivalent to GARP (Rose, 1958). This implies that when $K = 2$, there cannot be choices that satisfy WARP but violate GARP which, of course, is possible for $K \geq 3$.[1] However, little is known about the relationship between the set of possible revealed preference relations and the dimension of the consumption space when $K \geq 3$.

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[1] For example, a relation which is cycle consisting of three elements can never be generated by revealed preference when $K = 2$.

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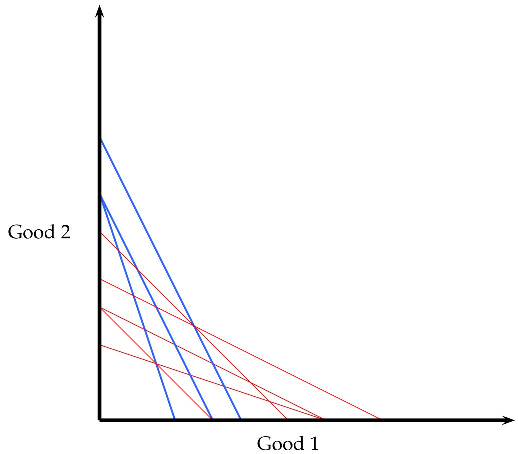

Fig. 1. Budget sets from Andreoni and Miller (2002).

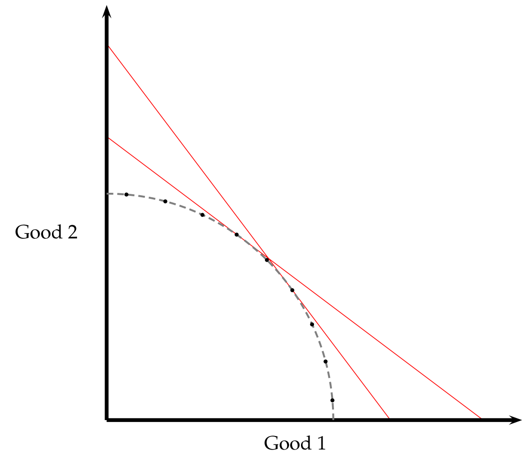

Fig. 2. Maximum WARP violations.

We show that when the dimension of the consumption space is large relative to the number of observations, revealed preference can generate any relation and conversely, certain rational and irrational preferences cannot be revealed by choices if the consumption space has low dimension relative to the number of observations.

As we will elaborate below, we feel that the results in this paper are useful for the design of experiments for models, the tests of which depend on revealed preference. A different but related problem which has received substantial recent attention is how to interpret the results of GARP tests. Suppose a researcher is interested in testing rationality on a given data set by checking for GARP violations. If the choices satisfy GARP, is it because the consumer is rational or is it because the budget sets provided little opportunity for rejecting rationality? Conversely, if the choices reject GARP, is there a simple way to interpret the degree of irrationality? These questions are addressed in a number of papers which have suggested both power (for example, Andreoni et al., 2013) and goodness of fit measures (for example, Beatty and Crawford, 2011) for a given collection of budget sets faced by a consumer. These measures are ideal to interpret results of GARP tests on a given price consumption data set.

By contrast, in experimental settings, the researcher is free to choose the budget sets, the dimension of the consumption space and the number of observations. Here, the experimental setup cannot be informed by the subject choices which are yet to be made at the design stage. As an example, consider the influential altruism experiment of Andreoni and Miller (2002). In this experiment, they varied relative prices and budgets and made individuals choose between keeping money for themselves and giving it to another subject. They then tested if charitable giving is rational. Fig. 1 shows the eight budget sets that they presented to their subjects. To provide a measure of the degree of irrationality of the subjects' choices, one of the statistics they reported was the number of WARP violations. They find that the choices of most irrational subjects contain only a single WARP violation (see Table 2 in their paper).

Suppose a researcher is interested in designing an experiment to study the irrationality of subjects measured by the number of WARP violations. What budget sets should be provided to the subjects? Presumably, the design should allow for choices which result in a large number of WARP violations. Clearly, the theoretical upper bound for the number of WARP violations in a subject's choices is achieved when every pair of her choices violate WARP. Fig. 2 shows that it is possible to provide subjects with budget sets on which such choices are possible. Here budget sets are chosen as different tangents to a given arc. Notice that if a subject's choices were the tangent point (or close to it) then every consumption bundle would be strictly revealed preferred to every other. In other words, every pair of such choices would violate WARP. Hence, by providing these budget sets to subjects, it is at least theoretically possible to observe choices with the maximal amount of irrationality. By contrast, consider the budget sets in Fig. 1 which were chosen by Andreoni and Miller. Note that the bold blue budget sets are such that any choices made on these budget sets will satisfy WARP. Hence, the most possible WARP violations that can be observed on these budgets sets are far fewer than the theoretical maximum.

Of course, testing GARP is just one instance of a test involving the revealed preference relation. Knowledge of the set of relations that can arise is important in experimental design for more general models as well. As an example, consider the design of an experiment to test the multiple rationale model of Kalai et al. (2002). In this model, an agent's preference depends on states. For instance, an agent may have different preferences depending on whether she is in a happy or a sad state. If these different states are unobserved by the researcher, then the observed choices from such an individual may be construed as irrational. Formally, in this setting, an individual has M rationales if there are M different states and the agent has a distinct utility function corresponding to each state. A data set is said to be "rationalized by M rationales" if each observed choice is the utility maximizing bundle corresponding to one of these preferences. The revealed preference test for M rationales simply involves partitioning the data into M subsets, such that GARP is satisfied on each of these subsets separately. A good experimental design for such general models should allow for the possibility that the hypotheses we want to test for can possibly arise from choices on the given budget sets. In particular, an important design choice is the number of goods in the subjects' choice sets.

As briefly mentioned above, our contribution is in the form of two theorems. In the first, we show that as long as the data set contains as many goods as observations ($K \geq N$), every possible binary relation can be generated using revealed preference. The proof is constructive and demonstrates how to generate a data set corresponding to a given relation. This can be viewed as a positive result for designing experiments which test properties of the revealed preference relation.

While the above result is positive, it suggests that there may be a connection between the number of goods and the nature of preferences that can be revealed by choices. Our second result shows that this is indeed the case. We prove that for every K, there exists a binary relation $\succ$ defined on a set with $N = O(2^K)$ elements such that there is no price consumption data set consisting of N