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Refinements and incentive efficiency in Walrasian models of insurance economies

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ABSTRACT

The literature on Walrasian markets in large economies with adverse selection has used various equilibrium refinements, but has obtained no general incentive efficiency of equilibrium, namely when cross-subsidies are needed for efficiency. We show that the same refined equilibria may also be incentive inefficient even when general *mechanisms* that allow for such cross-subsidies are priced and can be traded. In the process, we also prove existence of some type of forward induction equilibria in this context.

1. Introduction

Rothschild and Stiglitz' (1976; hereafter, RS) famous analysis of competitive screening in insurance economies leads to a nonexistence result that cast doubts on the viability of competitive markets with adverse selection. Some prominent papers (e.g., Gale, 1992, Zame, 2007; Dubey and Geanakoplos, 2002; hereafter DG) have since argued that Walrasian markets actually can function in the presence of adverse selection: an equilibrium for suitably defined competitive markets actually exists. The modeling strategy undertaken has defined either a market for contracts (Gale's approach) or for pools (DG's approach), and then applied refinement notions reminiscent of some criteria found in the game theoretic literature on Nash equilibrium in incomplete information games—stability, 'optimistic beliefs', or population perfection.[1]

The described Walrasian market systems, though, price only contracts or pools. It is then natural to ask what the market system misses from not pricing instead general mechanisms, that is, menus of contracts. This issue is not secondary as refined equilibria of the above-mentioned Walrasian markets do not guarantee the selection of the (constrained) Pareto optimal outcome. Indeed, the refined competitive equilibrium found in these models mimics the RS separating allocation—even when the latter is not constrained efficient. Competitive equilibrium fails to be constrained efficient when efficiency requires cross-subsidies. Mechanisms instead generally allow for a variety of transfers across agents, even of different types. It could be argued that the reason for the inefficiency lies in what these market systems price: by pricing only contracts or pools, these price systems would not give cross-subsidies enough of a chance.

In this paper, we extend Gale's model to let agents trade mechanisms, as opposed to contracts, thereby allowing explicitly for the possibility of cross-subsidization. With his stability criterion retained, we show that what amounts essentially to the RS separating contract is still in the stable set even when it is not constrained efficient. In fact, we prove this by using an optimistic perturbation of the agents' beliefs, in the DG sense, therefore also showing that the optimistic belief refinement cannot rule out the no-cross-subsidy separating allocation even when such cross-subsidies are explicitly allowed. Since we only need to prove that this happens in an open set of economies which includes the interesting cases dealt with by the previous literature, we focus our argument on the canonical example of the RS insurance economies.

All the above-listed refinements fail to display incentive efficiency for the same reason. We allow any mechanism to be traded, whether budget balanced or not: in Walrasian markets prices clear markets and enforce feasibility, or material balance, and there is no reason to rule out mechanisms from trading only because they are not budget balanced, i.e., feasible. With the inclusion of unfeasible mechanisms in the market, there are mechanisms that, though not necessarily feasible, are preferred over a given contract only by the

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[1] Gale (1992) uses essentially Kohlberg and Mertens' (1986) notion of stability; DG instead introduce a notion of refinement under optimistic belief perturbations; Zame (2007) applies a notion of population perfection.

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better types. These can be thought of as 'cream-skimming' mechanisms, although it is worth stressing that here no firm is actively reacting and trying to steal good types from other firms: instead, firms are price takers and simply offer slots at mechanisms, given those prices. For incentive efficiency to obtain, prices must adjust at equilibrium to dissuade good types from choosing the cream-skimming mechanisms. However, in all these refinements, prices must reflect the good-types odds; therefore, prices are not high enough to discourage the purchase of cream-skimming mechanisms by good types, and to sustain incentive efficient mechanisms – other than the RS separating contracts – as equilibrium outcomes.

Because everything is mediated here by prices, taking into account the zero-profit constraint, traditional 'cream-skimming' arguments must be considerably adapted. By the same token, we also prove in passing the existence of forward induction equilibrium in these Walrasian markets, a novel technical contribution of this paper. To wit, the strategic literature deals with quasi-linear utilities, whereas here utilities are not restricted to this class. Forward induction equilibria have also been used in the related competitive search with adverse selection literature, spearheaded by Gale (1996) and substantially completed by Guerrieri et al. (2010). Guerrieri et al. (2010) also have shown that using mechanisms does not lead to incentive efficiency in a forward equilibrium. However, Gale (1996) and Guerrieri et al. (2010) deal with a competitive matching model where probabilities, and not prices, clear markets, hence their existence proof does not apply here. We elaborate on this point later on in the paper.

In the simple environments of RS insurance economies studied here, Miyazaki (1977) first showed that the unique equilibrium of strategic Wilsonian competition over mechanisms is the constrained Pareto efficient outcome preferred by the high-quality type, and never includes the Rothschild and Stiglitz separating outcome when it is not efficient. Miyazaki's analysis was carried out in the absence of a Walrasian market—where a Wilsonian equilibrium notion is not straightforward and has never been introduced. Thus, this paper is the first step in the analysis of the efficiency of Walrasian market equilibria. The potential inefficiency of equilibria in the stable set points in the direction of a second step, combining Wilson equilibria and mechanisms, the subject of related work (see Citanna and Siconolfi, 2013).

The paper is organized as follows. Section 2 introduces the model, and defines the standard notions of (direct) mechanism. Section 3 introduces a price system, i.e., a Walrasian market for (lotteries over) mechanisms. Section 4 then defines a competitive equilibrium, and compares this competitive equilibrium with the related literature. Section 5 defines feasibility and incentive efficiency. Section 6 discusses some preliminary properties, while Section 7 is devoted to the analysis of refinements in the market for mechanisms. After the Conclusions, the Appendix contains most proofs.

2. The economies

We look at the simplest formulation of a large insurance economy with asymmetric information. There is only one physical consumption good, and a continuum of individuals with two unobservable types $s \in S = \{b, g\}$. We denote by π_s the fraction of the type s agents in the population, with $\pi_s > 0$ and $\sum_{s \in S} \pi_s = 1$. Individuals have type-invariant, uncertain endowments, subject to two idiosyncratic shocks $\omega \in \Omega = \{L, H\}$. The individual endowment is e_ω with $e_H > e_L > 0$. There is an exogenously given and commonly known type-dependent probability distribution over idiosyncratic shocks $\pi(\omega|s)$. It is $\pi(H|g) > \pi(H|b)$. Therefore, $\mathbb{E}_g(e) > \mathbb{E}_b(e)$, where $\mathbb{E}_s(\xi) = \sum_\omega \pi(\omega|s)\xi_\omega$ for any random variable ξ on Ω.

Individual preferences are represented by a von Neumann–Morgenstern utility function with type invariant Bernoulli index $v : \mathbb{R}_+ \to \mathbb{R}$, a continuous, strictly increasing and strictly concave map. Thus, the utility to a type-s individual generated by a net trade $z \equiv (z_\omega)_{\omega \in \Omega} \geq -e \equiv -(e_\omega)_{\omega \in \Omega}$ is

$$u(z, s) \equiv \mathbb{E}_s[v(z + e)].$$

State invariance of the cardinality index and $\pi(H|g) > \pi(H|b)$ are often referred to as the single-crossing property in this context.

Mechanisms

A contract, z, is a state-contingent net trade, $z = (z_L, z_H)$. A (direct) mechanism ζ is an insurance menu (pair) of contracts. We index the two contracts in ζ by $s = b, g$, so that $\zeta = (z_b; z_g) = ((z_{Lb}, z_{Hb}); (z_{Lg}, z_{Hg}))$. Individual trades are assumed to be fully verifiable and enforceable, i.e., contracts are exclusive. As well known, we can restrict attention to deterministic mechanisms because with type-invariant utility indexes, incentive efficiency does not require randomizations.

We denote the set of mechanisms by Z. A type-s individual preferences for a mechanism ζ are

$$U_s(\zeta) = \max_\sigma \mathbb{E}_s[v(z_\sigma + e)]$$

reflecting the fact that individuals can hide their type and choose among contracts. In principle the set of mechanisms Z is very rich, that is, it contains mechanisms, budget balanced or not, incentive compatible or not. Without loss of generality we can restrict attention to (or define the set of mechanisms as) the set X of *pairs* of *incentive compatible* contracts, that is,

$$X = \{\zeta \in Z : \mathbb{E}_s[v(z_s + e)] \geq \mathbb{E}_s[v(z_{s'} + e)], \text{ all } s, s' \in S\}.$$

With some abuse of notation we also write

$$\mathbb{E}_s(\zeta) = \sum_\omega \pi(\omega|s) z_{\omega s}$$

for the expected net resources consumed by type s in mechanism ζ. The set X is taken to be compact, and will be further specified below. Notice how the space of *contracts* is identified here with the subset of X consisting of *pooling* mechanisms, i.e., ζ with $z_b = z_g$.

3. A price system

Let $\Delta(X)$ be the set of lotteries μ over X.[2] We allow agents to buy lotteries $\mu \in \Delta(X)$ over slots at mechanisms in X, at prices $p(\mu)$. We introduce lotteries not because they help achieving efficiency, but because we want our prices to be linear in the objects of trade. In other words, trading lotteries is a way to make markets Walrasian.

The utility from a lottery $\mu \in \Delta(X)$ is

$$U_s\mu = \int_X U_s(\zeta) d\mu(\zeta).$$

For any mechanism ζ, let δ_ζ be the *degenerate* lottery assigning probability one to the singleton $\zeta \in X$ (δ is the Dirac function).[3] Of course, an agent can buy the degenerate lottery δ_ζ at $p(\delta_\zeta) \equiv p(\zeta)$. One can think of $p(\zeta)$ as the fee paid by an individual to participate mechanism ζ—the price of a ticket to enter ζ. If this number is negative, the agent receives money – more precisely, units of account, which do not enter the utility function – to enter the mechanism. Crucially $p(\zeta)$ is paid before entering the mechanism and choosing one of its components.

[2] Whenever needed, hereafter we assume that $\Delta(X)$ is a subset of $ca(X)$, the set of signed Borel countably additive measures over X of bounded variation, endowed with the weak* topology. Because X is compact, metric, $\Delta(X)$ is then weak* compact and metrizable.

[3] Since $U_s\delta_\zeta = U_s(\zeta)$, we feel free to switch from one to the other notation whenever more convenient.