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Trade through endogenous intermediaries☆

Weerachart T. Kilenthong[a,*], Cheng-Zhong Qin[b]

[a] University of the Thai Chamber of Commerce, Bangkok, 10400, Thailand
[b] University of California, Santa Barbara, CA 93106, USA

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ABSTRACT

We propose an intermediation core for an economy that explicitly specifies how traders organize themselves into trade cooperatives (intermediaries) and how trade between them gets carried out. The intermediation core allocations are closely related to the equilibrium allocations of a non-cooperative intermediation game in Townsend (1983). We show that the intermediation core contains all subgame perfect equilibrium allocations of the intermediation game, similar to the inclusion of competitive equilibrium allocations in the core usually studied. We identify intermediation core allocations that are also subgame perfect equilibrium allocations of the intermediation game in terms of the supporting intermediary structures. These results help to characterize subgame perfect equilibrium allocations of the intermediation game and to analyze their welfare and stability properties.

1. Introduction

The *core* of an exchange economy is based on coalitional rather than individualistic improvements that depend on what each coalition can achieve with its own members. The usual core is based on the assumption that any reallocation of coalition's total endowment among its members is feasible for the coalition. However, it is unclear how members organize themselves into the coalition and how they carry out the trade.

This paper has two purposes. First, we use the idea of intermediation to explicitly specify how economic traders organize themselves into trade cooperatives and how trade between them is carried out. This calls for the reformulation of what would be feasible for a coalition of traders to achieve. In this paper, an allocation is feasible for a coalition of traders if one of them acts as an *intermediary*, offering to buy and sell at a price vector, while the others act as *price-taking customers*. At each feasible allocation of a coalition, all members, possibly except for the intermediating trader, maximize their utility subject to budget constraints. For an allocation to be feasible for the economy, however, we allow for the possibility that trade is carried out by multiple disjoint intermediaries. The core resulting from this formulation of coalitional feasible allocations will be referred to as the intermediation core.[1]

Second, we relate intermediation core allocations with subgame-perfect equilibrium allocations under the approach taken in Townsend (1983). This approach provides an opportunity for each trader to play the role of an intermediary. One formalization of the approach works as follows. In the first stage, each trader individually and simultaneously offers to buy or sell commodities at a certain price vector and for a certain group of customers, subject to feasibility constraints. A trader may be offered a membership to multiple intermediaries. However, each trader must subsequently choose to trade with at most one intermediary in the second stage. Furthermore, a trader is obligated to intermediate under the announced terms should some of his potential customers choose to trade with him. Otherwise, he is free to act as the customer of an intermediary that includes him as a customer.[2] Because a trader's second-stage feasible choices

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* Corresponding author. Tel.: +66 26976637.
E-mail addresses: tee@riped.utcc.ac.th (W.T. Kilenthong), qin@econ.ucsb.edu (C.-Z. Qin).

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[1] Feasible coalitional allocations in this paper are different from those in both Mas-Colell (1975) and Qin et al. (2006). In the former, the feasible allocations of a coalition are required to be in competitive equilibrium of the sub-economy composed of members of the coalition, whereas in the latter, no one is required to maximize utility subject to budget constraints.

[2] This is one of the several variants of the model in Townsend (1983). See Townsend (1978), Boyd and Prescott (1986) and Boyd et al. (1988) for applications of the intermediation games.

depend on the choices of other traders, the *social equilibrium* in Debreu (1952)[3] is applied to the subgames in the second stage in the determination of a subgame perfect equilibrium (SPE).

An SPE of the intermediation game has the following properties. First, each non-intermediating trader maximizes utility by choosing an intermediary to trade with as well as the trade amount. Second, traders divide themselves into disjoint trading cooperatives, such that there is an active intermediating trader within each cooperative who specifies the terms of trade. Third, trade is stable in the sense that there is no entry of new intermediaries or exit of existing ones.[4]

We show that SPE allocations of the intermediation game are contained in the intermediation core under general conditions, similar to the inclusion of competitive equilibrium allocations in the usual core. We identify intermediation core allocations that are also SPE allocations in terms of the supporting intermediary structures. It is shown that an intermediation core allocation can be decentralized as an SPE allocation, whenever all intermediaries in the supporting intermediary structure have at least two customers. This stability of the intermediation core allocations resembles the *contestability* concept found in the industrial organization literature (Baumol et al., 1982). In particular, the two-customer requirement ensures that, for any active intermediary, there are always at least two contestable intermediaries that are ready to serve the other customers under the same terms.

Our paper also contributes to the literature on implementation of social choice correspondences using extensive form mechanisms and subgame-perfect equilibrium as a solution concept.[5] A group of papers in this literature considers subgame-perfect implementation of cooperative game-theoretic solutions. For example, Serrano and Vohra (1997) and several others consider the core usually studied as the social correspondence and its subgame-perfect implementation. In a similar spirit to theirs, viewing the intermediation game (or the rules of the intermediation game) as an extensive form mechanism, our results establish a subgame-perfect implementation of intermediation core allocations that are supportable by intermediaries with two or more customers.

The remainder of the paper is organized as follows. Section 2 introduces the intermediation core, intermediation game, and subgame perfect equilibrium of the game. Section 3 establishes the main results and Section 4 concludes. Appendix A contains proofs of theorems, and Appendix B presents an example of an unequal treatment of the intermediation core.

2. Intermediation in an exchange economy

Let $N = \{1, 2, \ldots, n\}$ be the set of traders and l ($<\infty$) be the number of commodities. Trader $i \in N$ has consumption set $X^i \subset \Re^\ell_+$ and initial endowment $\omega^i \in X^i \cap \Re^\ell_{++}$. His preferences can be represented by an increasing utility function $U^i : X^i \to \Re$. An exchange economy is described by the list $\mathcal{E} = \left(X^i, U^i, \omega^i\right)_{i\in N}$.

2.1. Intermediation core

The core concept is based on what players can achieve by organizing themselves into coalitions. For the usual core of an exchange economy, any allocation satisfying

$$\sum_{i\in C} x^i = \sum_{i\in C} \omega^i, \quad x^i \in X^i,\ i \in C \tag{1}$$

is regarded as feasible for traders in coalition C. This feasibility condition does not explicitly specify how traders organize themselves into coalition C and how trade between them is carried out. In this paper, we make the organization of traders into a coalition and trade between them explicit by requiring one of these traders to intermediate for the rest of them.

Definition 1. Given $C \subseteq N$, a C-allocation $\left(x^i\right)_{i\in C}$ is *feasible* for coalition C if it satisfies (1) and there exists a price vector p such that x^i solves

$$\max_x U^i\left(y^i\right) \text{ subject to } p \cdot x = p \cdot \omega^i, \quad x \in X^i \tag{2}$$

for all $i \in C$ except for at most one member j in C, in which case j receives bundle $x^j = \sum_{i\in C} \omega^i - \sum_{i\in C_{-j}} x^i$ with $C_{-j} = C \setminus \{j\}$.

The set of all C-feasible allocations is denoted by $F(C)$. The trader whose bundle does not maximize utility subject to budget constraints at an allocation in $F(C)$ is the intermediating trader. The remaining members are the customers of the intermediary. If each member's bundle maximizes utility subject to budget constraints, any one of them can be the intermediating trader.

Definition 2. An allocation $x^* = \left(x^{*i}\right)_{i\in N}$ is in the intermediation core if there is a partition $\{C^{*k}\}_{k=1}^m$ of N such that $(x^{*i})_{i\in C^{*k}} \in F(C^{*k})$, for $k = 1, 2, \ldots, m$, and there is no coalition $C \subseteq N$ and $(x^i)_{i\in C} \in F(C)$ such that $U^i(x^i) > U^i(x^{*i})$ for all $i \in C$.

Given an intermediation core allocation x^*, we call the collection $(p^{*k}, C^{*k})_{k=1}^m$ a supporting intermediary structure for the allocation x^* if, for each coalition k, price vector p^{*k} supports allocation $(x^{*i})_{i\in C^{*k}} \in F(C^{*k})$. Note that to be in the intermediation core, we allow for an allocation of the economy to be achievable through multiple disjoint intermediaries in stead of just one grand intermediary. The intermediation core remains the same if for any coalition C, we modify $F(C)$ by allowing trade between members in coalition C to be achievable though multiple disjoint intermediaries. The reason for this is that if a partition of C can improve upon a given allocation, then any sub-coalition in the partition can also improve upon the allocation.

The following example illustrates that the intermediation core of an economy is not included in its usual core.

Example 1. Consider an exchange economy with two commodities and three traders. The traders' endowments are $\omega^1 = \omega^2 = (6, 0)$ and $\omega^3 = (0, 12)$. Their utility functions are $u^i(x^i) = x^i_1 x^i_2$ for $x^i \in X^i = \Re^2_+$, $i = 1, 2, 3$. The allocation $\bar{x} = (\bar{x}^1, \bar{x}^2, \bar{x}^3)$ with $\bar{x}^1 = (3, 1) = \bar{x}^2$, and $\bar{x}^3 = (6, 10)$ is not in the usual core of this economy because it is not Pareto optimal.

However, $\bar{x}$ is in the intermediation core. Consider the following supporting intermediary structure. Let trader 3 be an intermediary with a price ratio $\rho = p_1/p_2$ and with traders 1 and 2 as customers. The demand of each customer is $(3, 3\rho)$. As a result, the allocation of the intermediary is $(12, 12) - 2 \times (3, 3\rho) = (6, 12 - 6\rho)$. It is clear that allocation $\bar{x}$ can be supported by price ratio $\rho = 1/3$. We will show that this allocation cannot be improved upon. It is clear that no individual trader alone can improve allocation $\bar{x}$.

We now consider grand coalitions. If trader 3 is an intermediary, then it is impossible to make traders 1 and 2 better off without making trader 3 worse off. If trader 1 is an intermediary with a price

[3] Yannelis (2009) generalizes this social equilibrium concept by allowing for asymmetric information and a continuum of agents.

[4] An existing intermediary with revised terms is regarded as a new intermediary.

[5] The interested reader is referred to Moore and Repullo (1988) and Abreu and Sen (1990), which were among the earlier papers in the literature.