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The effect of material and thickness variability on the buckling load of shells with random initial imperfections

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Abstract

The effect of material and thickness imperfections on the buckling load of isotropic shells is investigated in this paper. For this purpose, the concept of an initial 'imperfect' structure is introduced involving not only geometric deviations of the shell structure from its perfect geometry but also a spatial variability of the modulus of elasticity as well as the thickness of the shell. The initial geometric imperfections are described as a two-dimensional uni-variate (2D-1V) stochastic field with statistical properties that are either based on an available data bank of measured initial imperfections or assumed, in cases where no experimental data is available. In order to describe the non-homogeneous characteristics of the initial imperfections, the spectral representation method is used in conjunction with an autoregressive moving average model with evolutionary power spectra based on a statistical analysis of the experimentally measured imperfections. In cases where no experimental results is available, the initial imperfections are assumed to be homogeneous and their impact on the buckling load is investigated on the basis of 'worst'-case scenarios with respect to the correlation length parameters of the stochastic fields. The elastic modulus and the shell thickness are described as 2D-1V non-correlated homogeneous stochastic fields, while the stochastic stiffness matrix of the shell elements is formulated using the local average method. The Monte Carlo Simulation method is used to calculate the variability of the buckling load, while for the determination of the limit load of the shell, a stochastic formulation of the elastoplastic and geometrically non-linear TRIC facet triangular shell element is implemented.

Keywords: Non-linear shell finite element; Random imperfections; Spectral representation; Autoregressive model; Evolutionary spectra

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1. Introduction

The buckling behaviour of shell structures is generally influenced by their initial imperfections, which occur during the manufacturing and construction stages. Thus, the analysis of imperfection sensitive shells has attracted the attention of many researchers in the past. Although these research efforts resulted in achieving predictions close to the experimental results, it was soon realized that the wide scatter in measured buckling loads of shell structures could only be approximated through modeling taking into account the randomness of the imperfect geometries. This variability of initial imperfections together with their pronounced influence on the load carrying capacity of shells has been proved to be responsible for the large scatter observed in the experimental results [1–4]. In addition to initial geometric imperfections, other sources of imperfections such as the variability of thickness, material properties, boundary conditions and misalignment of loading are also responsible for the reduction as well as the scatter of the buckling load of shell structures [2,3,5]. In the majority of studies these influencing parameters have not been treated as stochastic variables in a rational manner. An accurate prediction of the buckling behaviour of shells would therefore require a realistic description of all uncertainties involved in conjunction with a robust finite element formulation that can efficiently and accurately handle the geometric as well as physical non-linearities of shell type structures [6].

In the present paper the effect of material and thickness imperfections on the buckling load of isotropic shells is investigated. For this purpose, the concept of an initial 'imperfect' structure is introduced involving not only geometric deviations of the shell structure from its perfect geometry but also a spatial variability of the modulus of elasticity as well as of the thickness of the shell. These combined 'imperfections' are incorporated in an efficient and cost effective non-linear stochastic finite element formulation of the TRIC shell element [8,9] using the local average method for the derivation of the stochastic stiffness matrix, while the variability of the limit loads is obtained by means of the Monte Carlo Simulation (MCS) procedure.

In order to investigate the influence of the material and thickness variability on the buckling load of shells with random geometric imperfections, two types of shell structures are selected with criterion their buckling behaviour up to the limit point. The first type is a shallow hinged isotropic cylindrical panel with a point load at the mid of its top surface. This shell exhibits a limit point buckling with large pre-buckling deformation response and considerable influence of the physical non-linearities on its buckling behaviour. The second type is a thin-walled isotropic axially compressed cylinder, which exhibits a bifurcation buckling that occurs while the structure remains elastic. Therefore, the second type is selected as an example of an imperfection-sensitive structure in the sense that small deviations from its perfect geometry may result in a dramatic reduction in its buckling strength [7], while the first type was selected in order to investigate the effect of material and thickness variability on the buckling behaviour of a less sensitive to initial imperfections type of shell.

The analysis of shell structures exhibiting physical and geometric non-linearities has received considerable attention over the past years and it has been shown the importance of both types of non-linearities on the carrying capacity of these structures. In this work a layered elastoplastic constitutive model based on the von Mises yield criterion, the associated flow rule and isotropic hardening is adopted in conjunction with the geometrically non-linear shell element TRIC [10]. The main advantage of this formulation is that the elastoplastic stiffness matrix is formed on the natural coordinate system and can be expressed analytically for each layer. Then, the total natural tangent stiffness matrix is computed by adding together the tangent stiffness matrix of each layer. This formulation inherits a number of advantages associated with the natural mode method. This elastic–plastic large displacement formulation is therefore considered a robust and cost-effective platform for the accurate prediction of the buckling and post buckling behaviour of imperfect shells.

The initial geometric imperfections are described as a two-dimensional uni-variate (2D-1V) stochastic field. In the case of the axially compressed cylinder, the statistical properties of the stochastic field are based