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Stochastic response of large FEM models with hysteretic behaviour in beam elements

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Abstract

An investigation of the hysteretic response of 3D frames in FE-systems with a high number of DOF under random dynamic load is presented. In stochastic analysis it is of high priority to provide a system of equations with a small number of unknowns. Working in a reduced modal base the contribution of the neglected higher modes is replaced by plastic shape functions. These shape functions are generated by a mechanical consideration of the local plastification effects.

The application of the different solution techniques on two large frame structures is discussed in the context of hysteretic moment–curvature relations of 2D and 3D flexible beams. The results are commented reflecting the accuracy and performance of the methods.

Keywords: Modal reduction; Plastic shape functions; Frame structures; Stochastic excitation; Hysteretic material laws

1. Introduction

Methods for the analysis of the nonlinear response with few degrees of freedom are well developed for a variety of stochastic excitations and nonlinear systems. However most of the presently used methods involve significantly higher numerical complexity and computational effort than in the case of deterministic calculations.

It is favourable if the same principal algorithms can be applied as in deterministic analysis. Monte-Carlo simulation methods admit the largest bandwidth of excitation types and nonlinear models [7,16,17]. Since

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the stochastic characteristics are transferred into a series of deterministic sample inputs, no further solution methods have to be developed. Approaches that directly process the stochastic input characteristics for a lot of cases are more elegant and often faster, however often imply significant restrictions and assumptions which only hold for a specific set of conditions.

The fundamental step to enhance the acceptance of stochastic investigations in practical problems is to apply effective reduction techniques to break down the system to a treatable size without considerable loss of information. For linear elastic calculations the modal reduction technique is the most common and effective procedure when dealing with structures in practise. If local external or internal nonlinear forces or constraints act on the system, higher modes have to be added to approximate the local deformations in superposition. To avoid an excessive extension of the modal base alternatively additional global shape functions that describe the high local load and deformations are introduced for further processing in the Galerkin sense. The shape functions were introduced as "plastic modes" or "plastic shape functions" [14,15]. The extension of the modal base by these shape functions has been detailed for shear beam models and continuous 2D-frame structures with flexible beam elements.

The introduction of additional shape functions within a dynamic analysis in a reduced modal base appears also in the modal acceleration method or modal augmentation method [13]. Nonstationary problems of nonlinear large systems have been solved very efficiently with this technique in [8]. The methods can also be considered in a very close connection to component synthesis method (CMS). The disadvantage of the latter is that the spots of plastification have to be known a priori. The extension of the modal augmentation method to local nonlinearities can be interpreted as a CMS procedure where the modes of components are subtracted from the total linear system instead of assembling the total system of modes of the different components.

In the examples the response of large FE-structures subjected to stochastic excitation is discussed under these specifications:

- Plastification develops within 2D and 3D flexible beam elements.
- The introduction of plastic form functions in connection with nonlinear moment–curvature relations in beam elements is detailed. It will be shown that the truncation error using only "linear" modes can become significant in the moment–curvature relation. Merging plastic form functions to the reduced modal base does not only reduce the truncation error but is capable to compensate the deficiency to a great amount and becomes essential for large systems.
- The spots of plastification are a priori unknown; the linear elastic modes are not recomputed or modified when new nonlinear equations to the system are appended to the system.
- The stochastic response characteristics are gained by Monte-Carlo simulation (2D and 3D) and equivalent statistical linearization (2D).
- All algorithms are implemented in a commercial FE-software. Priority is given to the application of routines and database information of large FE-models provided by the software.

2. Nonlinear moment–curvature relations

If the hysteretic restoring moment can be described by a nonlinear function in the endochronic form of

$$\dot{M}_{\mathrm{H}} = G(\kappa, \dot{\kappa}, Y_{\mathrm{H}}, \dot{Y}_{\mathrm{H}}), \tag{1}$$

the restoring moment is split up into two additive contributions:

$$M = M_{\mathrm{el}} + M_{\mathrm{H}} = -EI\kappa \cdot \alpha - EI\kappa_{\mathrm{Y}} Y_{\mathrm{H}}(1 - \alpha). \tag{2}$$