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Boundary element analysis of three-dimensional mixed-mode cracks via the interaction integral

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Abstract

A three-dimensional boundary element method (BEM) implementation of the interaction integral methodology for the numerical analysis of mixed-mode three-dimensional cracks is presented in this paper. The interaction integral is evaluated from a domain representation naturally compatible with the BEM, since stresses, strains and derivatives of displacements at internal points can be evaluated using their appropriate boundary integral equations. Special emphasis is put in the selection of the auxiliary function that represents the virtual crack advance in the domain integral. This is found to be a key feature to obtain reliable results at the intersection of crack fronts with free surfaces. Several examples are analysed to demonstrate the efficiency and accuracy of the implementation.

Keywords: Fracture mechanics; Energy domain integral; Interaction integral; Boundary elements

1. Introduction

Evaluation of fracture parameters is usually the purpose of carrying out a numerical model of a crack problem. Two-dimensional analyses are usually sufficient to characterize through-thickness cracks. However, part through cracks, which are the most common type of crack defect found in service conditions, have an inherently three-dimensional character. The solution to three-dimensional crack problems can

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be obtained by such techniques us the finite element method (FEM) and the boundary element method (BEM).

When the basic assumption of linear elastic fracture mechanics is adopted, the stress intensity factors can be evaluated in FEM and BEM by a variety of techniques, such us the extrapolation of displacements or stresses, special crack tip elements, the virtual crack extension method, the subtraction of singularity technique, the alternating method and J-integral methods [1,2]. Techniques based on the extrapolation of displacements an stresses are easy to implement, but they present the drawback that a very high level of mesh refinement is required for its accurate evaluation [2], what makes them computationally expensive. Similarly, alternating and virtual crack extension methods are also computationally expensive, as they require of multiple computer runs to solve the problem. On the other hand, J-integral methods, being an energy approach, eliminate the need to solve local crack tip fields accurately, since if integration domains are defined over a relatively large portion of the mesh, accurate modelling of the crack tip is unnecessary because the contribution to J of the crack tip fields is not significant. The J-integral as devised by Rice [3] characterizes the crack driving force for two-dimensional problems, therefore for general three-dimensional cases involving cracks of arbitrary shape an alternative form for J is needed. Three basic schemes have evolved for the numerical computation of the J-integral in three dimensions: virtual crack extension methods [4,5], generalization of Rice's contour integral [6] and domain integral methods [7–9].

The BEM is ideally suited for the evaluation of path independent integrals, since the required stresses, strains and derivatives of displacements at internal points can be directly obtained from their boundary integral representations. It also has been shown that BEM produces more accurate stresses and strains at internal points when compared with other numerical techniques, and therefore better results can be achieved. The application of the BEM for the evaluation of J-integral in three-dimensional problems have been the reported by Rigby and Aliabadi [10], Huber and Kuhn [11], Cisilino et al. [12,13] and dell'Erba and Aliabadi [14]. Although the bulk of fracture mechanics literature is concerned with the first mode of crack deformation, there are practical engineering problems that involve mixed-mode conditions. Of the above cited papers, the works by Rigby [10] and dell'Erba [14] proposed methods for decoupling the J-integral for mixed-mode cracks in which the symmetric and antisymmetric parts of the displacement, strain and stress fields are separated.

Among the available methods for calculating fracture parameters, the energy domain integral (EDI) has shown well suited for three-dimensional BEM analysis [12,13]. The EDI is versatile, efficient and relatively simple to implement numerically. To develop the domain integral the EDI incorporates an auxiliary function q, which can be interpreted as a virtual crack front advance. This makes the EDI similar to the virtual crack extension technique [15,16] but has the advantage that only one computer run is necessary to evaluate the point wise energy release rate along the complete crack front. On the other hand, the interaction or M_1-integral methodology originally proposed by Chen and Shield [17] has emerged as an efficient methodology for decoupling the J-integral for mixed-mode cracks. The M_1-integral methodology is based on the superposition of two equilibrium states, given by the actual problem and a set of auxiliary known solutions. The M_1-integral methodology has been implemented using BEM for two-dimensional cracks by Miyazaki et al. [18].

This work presents a BEM domain formulation of the M_1-integral for the computation of mixed-mode stress intensity factors along three-dimensional crack fronts. The domain representation of the interaction integral is presented in a straightforward approach, together with the details of its BEM implementation. Special emphasis is put in the appropriate selection of the auxiliary function q, which was found to be a key feature to obtain reliable results at the intersection of crack fronts with free surfaces. Several examples are analysed to demonstrate the efficiency and accuracy of the implementation.