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Efficient implementation of the convective terms in the hydrodynamic equations

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Abstract

A new discretization scheme for the current density in the hydrodynamic model is presented. This scheme is an extension of the Scharfetter–Gummel (S–G) method and is derived without neglecting the convective term in the current density. Simulation of two model n–i–n diodes shows that this method gives better numerical results than the conventional upwind and S–G methods.

Keywords: Hydrodynamic model; Numerical simulation; Convective term; Scharfetter–Gummel method

1. Introduction

The electron gas in a semiconductor behaves as a compressible gas of charged particles in an electric field. The hydrodynamic (HD) equations are powerful tools for studying the electron gas transport in semiconductor devices. These equations are non-linear conservation laws for particle number, momentum and energy density. In semiconductor devices where active regions are well above one micron, electron temperature in the device is not much different from the ambient temperature. Therefore only balance equations for particle number and momentum is used, which gives rise to the drift–diffusion (DD) model. But in submicron devices, the DD model has not sufficient accuracy to describe the device operation. In these devices the balance equation for the energy density, in addition to the DD equations, must be used [8,17]. A simplifying approximation in the HD equations is to neglect the convective terms in the momentum and energy

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balance equations [9]. In this approximation, balance equations for particle number and momentum are merged together into a second order differential equation. This model is called energy balance model (EBM) [7].

In order to obtain a numerical solution for the system of HD equations, the balance equations must be discretized. For the DD model, the Scharfetter–Gummel (S–G) method is a successful method that prevents the formation of wiggles in the solution [14]. In this method, the balance equation for the momentum is solved, in the interval between two adjacent points of the mesh, as a first order differential equation for the carrier density in order to find a discretized form for the current density. This method results in an exponential form for the carrier concentration on each mesh sub-interval.

Since the S–G method has improved convergence and stability behavior, this method has been extended to the EBM. Several methods have been proposed for extending the S–G method to the EBM [2,15] and have been used for simulation of various electron devices. The S–G method and its extensions have desirable convergence and stability [2].

In devices with active regions well below one micron, electron velocity may reach values much above the saturation velocity. In these situations, the convective terms have significant values and neglecting them may cause noticeable error in the simulation results [10]. Also at low temperatures convective terms may not be negligible [11]. In these situations the HD equations (without neglecting convective terms) must be used.

Besides the S–G method and its extensions, the second upwind method [4] is a discretization scheme, which is consistent with the physics of the transport process [12].

In this paper, our goal is to extend the S–G method to the HD equations. The outline of this paper is as follows. In Section 2 the HD equations are introduced. In Section 3 the new method for discretizing the momentum equation is proposed. This method is derived without neglecting the convective term in the current density. In Section 4, some limiting cases are discussed and compared with existing schemes in the literature. The extension of the scheme to the multi-dimensional case is presented in Section 5. In Section 6, numerical simulation of two silicon n–i–n diodes is presented. We compare our method with the upwind and the conventional S–G method. Simulations show that the new method gives better numerical results for both long channel and short channel diodes. Finally, in Section 7 the main conclusions of the paper are presented.

2. The hydrodynamic equations

The time independent hydrodynamic equations in one dimension are [6]:

$$\partial_x(nv) = 0, \tag{1}$$

$$\partial_x(nv^2 + P) + nE + \frac{\alpha}{\tau_p} nv = 0, \tag{2}$$

$$\partial_x(nvw + vP) - k_0\gamma\partial_x(n\partial_x T) + nvE + \frac{\beta n}{\tau_w}\left[\frac{3}{2}(T-1) + \frac{1}{2}v^2\right] = 0, \tag{3}$$

$$w = \frac{3}{2}T + \frac{1}{2}v^2, \qquad P = nT, \tag{4}$$

where $E = -\partial_x\psi$ is the electric field and ∂_x denotes the derivative with respect to x. For self-consistent solution, Poisson's equation for the electric potential is also considered:

$$\nabla^2\Psi + \lambda(n_d - n) = 0. \tag{5}$$