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Numerical analyses of discontinuous material bifurcation: strong and weak discontinuities

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Abstract

In this paper an algorithmic formulation for numerical analyses of material bifurcation is presented. Conditions for the onset of both weak discontinuities (discontinuous strain rates) and strong discontinuities (discontinuous velocity fields) are summarized. Based on a recently proposed plasticity model formulated within the logarithmic strain space, the condition for the formation of strong discontinuities is extended to anisotropic finite strain plasticity theory. The resulting equations associated with the mode of bifurcation are solved numerically. For that purpose, an equivalent optimization problem is considered. The algorithmic formulation is based on Newton's method using a consistent linearization. To enlarge the radius of convergence, a line search strategy is applied. The applicability of the proposed implementation as well as its performance and numerical robustness is investigated by means of three-dimensional numerical bifurcation analyses of a Drucker–Prager type plasticity model.

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1. Introduction

For a better understanding of the mechanical behavior of materials, the knowledge about their corresponding failure modes is of utmost importance. For instance, the mode I failure associated with concrete subjected to tensile loading leads to a sudden drop in the residual strength of the material. In contrast to

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this, shear bands and slip lines (mode II failure) typically appear in metals and in saturated soils. In general, these mechanisms show a more ductile material response.

From a macroscopic point of view, material failure is often accompanied by the formation of narrow zones with high gradients of the displacement field. For impressive illustrations of such localized deformation fields, we refer to [1]. In the limit case, the gradient of the displacement field (GRAD$\boldsymbol{u}$) is discontinuous (weak discontinuity). The condition for the transition from homogeneously distributed deformations (see Fig. 1a) to discontinuous strain fields (see Fig. 1b) has been implicitly developed by Hadamard [2]. Hadamard analyzed the dispersion of waves in solids and derived the condition for real valued wave speeds. Not until the 1970s the connection between Hadamard's work and the transition from homogeneously distributed deformations to localized failure was understood. Rudnicki and Rice [3,4] studied this transition for different materials and computed the corresponding bifurcation modes. Raniecki and Bruhns [5] extended the ideas of Rudnicki and Rice to non-associated finite strain plasticity. For the geometrically linear theory explicit formulas for different plasticity based continuum models have been proposed in [6,7]. The counterparts for damage mechanics have been given in [8].

If the width of the zones exhibiting localized deformations converges to zero (see Fig. 1), a discontinuity in the displacement field forms. This limit case is often denoted as strong discontinuity. The conditions for the onset of discontinuous displacement fields have been analyzed by Simo et al. [9]. Before bifurcation, Simo et al. assumed a homogeneously distributed deformation field. The extension to finite strain plasticity was proposed in [10]. In contrast to [9,10], Oliver studied the transition from weak to strong discontinuities [11]. On the basis of an evolution law for the width of the zone in which localized deformations appear, the connection between both types of material bifurcation was achieved. The corresponding extension to finite strains was presented in [12]. However, both finite strain models [10,12] are based on a space of admissible stresses formulated in terms of Kirchhoff stresses together with an isotropic softening evolution (and without structural tensors). Consequently, they are restricted to isotropic yield surfaces. In this paper, we extend the ideas proposed in [10] to anisotropic finite strain plasticity theory.

The conditions associated with the formation of weak and strong discontinuities result in sets of equations formally identical to each other (see [10]). However, closed form solutions are not available for more

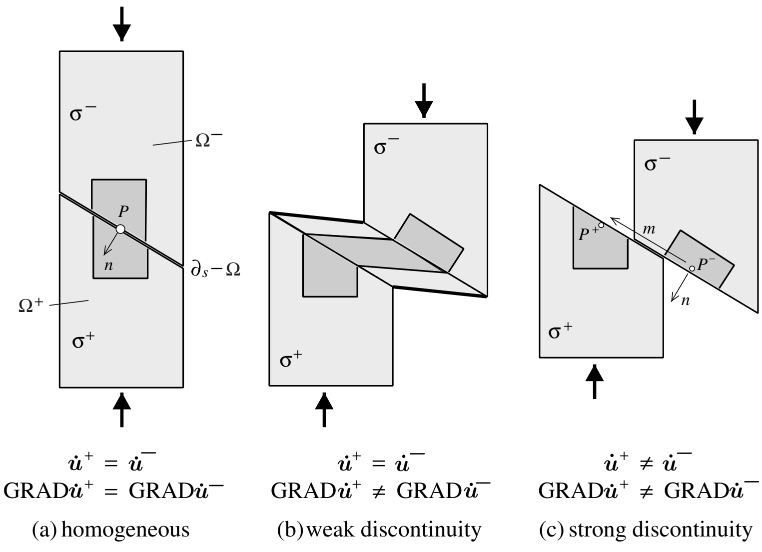

Fig. 1. Different stages of the formation of a shear band observed in soils (uniaxial compression test): (a) homogeneous, (b) weak discontinuity, and (c) strong discontinuity.