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Solving inverse problems involving the Navier–Stokes equations discretized by a Lagrange–Galerkin method

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Abstract

In this article, we are investigating the numerical approximation of an inverse problem involving the evolution of a Newtonian viscous incompressible fluid described by the Navier–Stokes equations in 2D. This system is discretized using a low order finite element in space coupled with a Lagrange–Galerkin scheme for the nonlinear advection operator. We introduce a full discrete linearized scheme that is used to compute the gradient of a given cost function by ensuring its consistency. Using gradient based optimization algorithms, we are able to deal with two fluid flow inverse problems, the drag reduction around a moving cylinder and the identification of a far-field velocity using the knowledge of the fluid load on a rectangular bluff body, for both fixed and prescribed moving configurations.

Keywords: Navier–Stokes equations; ALE formulation; Lagrange–Galerkin; Characteristics; Inverse problem; Quasi-Newton methods; Optimal boundary control of the tracking type

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1. Introduction

The aim of this paper is to deal with an identification problem arising in the design of bridge decks under wind loads. A rather good mechanical model for such an aeroelastic system is to consider a 2D rigid section surrounded by a 2D incompressible flow driven by the Navier–Stokes equations [33]. Our identification problem consists in recovering information about the inflow velocity field from the knowledge of the fluid

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loads around the bridge section. To our knowledge, except in [2] where an inflow inverse problem is solved in the case of a 2D supersonic flow driven by the parabolized Navier–Stokes equations, this problem has never been investigated before for the Navier–Stokes system. This inverse problem is handled using a least-square optimization method which is similar to an optimal control problem of the tracking type with inflow boundary control.

In the last two decades, number of studies concerning optimal control problems for fluid mechanics have been conducted. On the theoretical point of view, significant advances have been realized in the derivation of infinite dimensional optimality conditions [1,17,23] and in controllability properties investigations [10,25,13]. With the increasing power of computing capabilities, numbers of numerical studies have been also performed [27,5,20,22,24] with the objective of controlling fluids by minimizing cost functions involving various quantities such as the drag over a fixed obstacle, the vorticity level in a given fluid region or the final state fluid pattern. Most of these works use finite difference schemes for the time discretization of the Navier–Stokes system coupled with finite difference or finite element space discretizations. In this article, we propose to discretize the nonlinear-advection operator involved in the Navier–Stokes system using the characteristics method introduced by [4]. This method is based on the computation of characteristics paths in order to approximate the material time derivative. The use of this algorithm introduces some technical difficulties addressed in this paper. To our knowledge, except in [9] where the optimal control of the parabolized Navier–Stokes system is studied using the characteristics method, this problem has never been investigated before for the full Navier–Stokes system. Once the discrete linearized system is established, its solution enters the computation of cost-function gradients involved in the optimization loop. Two different applications are addressed:

- One concerning the drag reduction around a rotating circular cylindrical body using its angular velocity. At low Reynolds numbers, the 2D case is good model since the wake flow remains 2D [36]. This problem has been already treated by several authors [22,24]. Since our primary goal is mainly to validate our approach by analysing its ability of dealing with nontrivial control problem benchmarks, we did not try to improve earlier results but instead check if the computation of the discrete gradient involving the linearized characteristics leads to a convergent procedure. This control problem is referred as a local inverse problem since the cost function only involves quantities localized on the boundary of the cylinder where the control is applied. Hence, in this case, an incomplete sensitivity strategy is also valid and the derivative of the fluid state can be neglected in the computation of the cost-function gradient [28,29].
- Then, we deal with the identification problem that may enter an aeroelastic stability tool described in [30]. It consists in trying to identify far-field boundary conditions from the knowledge of fluid loads time history on a given bluff body in fixed or prescribed moving configuration. The final application should be the aeroelastic stability analysis of civil engineering super-structures [33,3,11]. This framework can also be used in order to solve data assimilation problems for wind tunnel tests. In this case, we deal with a nonlocal inverse problem where the observation of the state is located on the boundary of the bridge deck whereas the unknown variable is located on the inflow boundary. For such a problem, the incomplete sensitivity analysis is no more valid, since the derivative of the fluid state with respect to the inflow boundary condition will contribute in a major part to the gradient of the cost function.

The remainder of the paper is organized as follows:

- In Section 2, we introduce the Navier–Stokes equations together with a minimization problem for a particular cost function of the tracking type. In this setting, the unknown variable is the fluid velocity located either on the solid boundary or on the inflow boundary. We furnish the continuous gradient of this cost function and describe the minimization strategy based on a quasi-Newton method.