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Numerical solution of eddy current problems in bounded domains using realistic boundary conditions

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Abstract

The aim of this paper is to analyze a finite element method to solve the low-frequency harmonic Maxwell equations in a bounded domain containing conductors and dielectrics, and using realistic boundary conditions in that they can be easily measured. These equations provide a model for the so-called eddy currents. The problem is formulated in terms of the magnetic field. This formulation is discretized by using Nédélec edge finite elements on a tetrahedral mesh. Error estimates are easily obtained when the curl-free condition is imposed explicitly on the elements in the dielectric domain.

A multivalued magnetic scalar potential is introduced then to impose this curl-free condition. The discrete counterpart of this formulation leads to an important saving in computational effort. Problems related to the topology are also considered, more precisely, the possibility of having a non-simply connected dielectric domain is taken into account. Finally, the method is applied to solve two three-dimensional model problems: a test with a known analytical solution and the computation of the electromagnetic field in a metallurgical arc furnace.

Keywords: Low-frequency harmonic Maxwell equations; Eddy current problems; Finite element approximation; Realistic boundary conditions

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1. Introduction

In this paper we analyze a finite element method to solve the eddy current model in a bounded domain including conducting and dielectric materials. This model can be obtained from Maxwell equations by assuming that all fields are harmonic and that the current frequency is low enough so that the term involving the displacement current in Ampère's law can be neglected. Such a situation happens, for instance, in problems related to machines working at power frequencies. In particular, this paper is motivated by the need for a three-dimensional numerical simulation of a metallurgical furnace. See [8–10] for related works and [6,7] for axisymmetric models.

Because of many interesting applications in electrical engineering, numerical solution of eddy current problems became an important research area, leading to a great number of publications in recent years. See, for instance, [2–4,12,14,16,18,20,23]. The books by Bossavit [13] and Silvester and Ferrari [22] contain valuable material on this subject and include large reference lists.

While most of these papers deal with the eddy current problem in the whole space, there are only a few ones concerning analysis in a bounded domain. This is due, in part, to the difficulty of handling realistic boundary conditions. For instance, essential and natural boundary conditions related to the tangential component of the electric and magnetic fields are considered in [1–3,8]. However, these conditions are not directly related to the physical data which, in the case of an electric furnace, usually reduces to the input current intensity through a part of the boundary of the conducting domain. A first step towards dealing with realistic boundary conditions has been done in [9,11], where a bounded conducting domain is considered and input current intensities or densities are imposed via Lagrange multipliers.

In the present paper we also consider a bounded domain, but one which includes conductors and dielectrics; the conductors are not supposed to be totally included in this domain. Following a model introduced in [15], we propose and analyze a finite element method to solve an eddy current problem involving boundary conditions appropriate from a physical point of view. In particular, we consider a formulation in terms of the magnetic field and impose boundary conditions related to the current intensities entering the conducting domain, by means of Lagrange multipliers. Then, following [16], we introduce a multivalued scalar magnetic potential in the domain occupied by the dielectric.

Concerning the discretization of the problem, the magnetic field is approximated by Nédélec edge finite elements and the magnetic potential by standard piecewise linear continuous elements. The current intensities are imposed as jumps of the multivalued magnetic potential on some prescribed cut surfaces.

The outline of the paper is as follows: In Section 2, we recall the eddy current model and define a set of boundary conditions under which the electromagnetic problem is well posed. Then, in Section 3, we obtain a weak formulation and prove existence and uniqueness of solution. In Section 4, we introduce a scalar magnetic potential in the dielectric domain. The numerical discretization is introduced in Section 5, where error estimates are obtained under mild regularity assumption on the solution. Implementation issues are also discussed. Finally, in Section 6, we report some numerical results including an application to a metallurgical furnace.

2. Eddy current problem with input current intensities as boundary data

Eddy currents are usually modeled by the low-frequency harmonic Maxwell equations:

$$\mathbf{curl}\,\boldsymbol{H} = \boldsymbol{J}, \tag{2.1}$$

$$\mathrm{i}\omega\mu\boldsymbol{H} + \mathbf{curl}\,\boldsymbol{E} = \mathbf{0}, \tag{2.2}$$