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Dynamic instability of rectangular plate with an edge crack

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Abstract

In this study, the dynamic instability and non-linear response of the cracked plates subjected to period in-plane load are theoretically analyzed. By applying the Galerkin's method to the dynamic analog of the von Karman's plate equations, the governing equation of a cracked plate is reduced to a time-dependent Mathieu equation. The incremental harmonic balance (IHB) method is applied to solve the non-linear temporal equation of motion and analyze the dynamic instability in this study. Calculations are carried out for the rectangular plates of various aspect ratios under different values of crack ratio conditions. Regions of parametric instability are presented in the spaces of the excitation load versus natural frequency and the natural frequency versus amplitude. In particular, the amplitude of vibration of the plate is considered as an additional parameter of the system. Therefore, the effects of various system parameters on the regions of instability and the non-linear response characteristics of rectangular cracked plates have been investigated and discussed in this study.

Keywords: Dynamic instability; Crack; Plate; von Karman; Mathieu equation; Incremental harmonic balance (IHB) method

1. Introduction

The dynamic instability of plate has received considerable attention in recent years. As is well known, when a plate subjected to a periodic in-plane force of the form $N(t) = N_0 + N_t \cos \varpi t$, the dynamic instability may occur over certain regions of the (N_0, N_t, ϖ) parameter space and the amplitude of transverse vibration may increase without bound. In the case of simple principal resonance that occurs when the excited frequency of the in-plane force is twice of the natural frequency or the frequency of transverse load equals to the natural frequency.

Detailed reviews of dynamic instability of various elastic systems and discussed the peculiarities of the phenomena of instability had been presented by Bolotin [1], Evan-Iwanowski [2], and Yamaki and Nagai [3]. Takahashi and Konishi [4] are considered the dynamic stability of a rectangular plate under a linearly distributed period load applied two opposite edges. Ostiguy and Evan-Iwanowski [5], and Nguyen and Ostiguy [6] considered the influence of the aspect ratio and boundary conditions on the dynamic instability and non-linear response of rectangular plates. Duffield and Willens [7] presented an analytical and experimental investigation of parametric instability for a stiffened rectangular plate. Therefore, the dynamic instability of the plate has already created the way for direct engineering application.

Since the current study presents a theoretical analysis of the effects of crack ratio on the stability characteristic and the non-linear responses of a cracked plate subjected to both static and period in-plane loads applied along two opposite edges. The crack ratio play a critical consideration in this study, the natural frequency of the cracked plate must be mainly confined to the investigation of principal instability region. The analyses of vibration for a cracked rectangular plate were investigated by Lynn and Kumbasar [8], and Stahl and Keer [9], who have solved the eigenproblems of simply supported plate by using homogeneous Fredholm integral equation. Maruyama and Ichinomiya

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[10] obtained experimentally the natural frequencies for rectangular plates with straight narrow slits, which the effects of lengths, positions and inclination angles of slits on natural frequency and mode shape are discussed. Hirano and Okazaki [11], Neku [12] and Solecki [13] have analyzed the internal crack problem by means of finite Fourier transformation. The discontinuities of the displacement and the slope of the crack are expanded into Fourier series then the characteristic equation in form of an infinite determinant is obtained. Yuan and Dickson [14], and Lee and Lim [15] used the Rayleigh–Ritz approach for the study of the free vibration of the system. Further the results of finite element method for dynamic analysis of a thin, rectangular plate with a through crack under bending, twisting and shearing have been formulated by Qian et al. [16], Krawczuk [17,18], and Krawczuk and Ostachowicz [19]. Liew et al. [20] employed the decomposition method to determine the vibration frequencies of cracked plates. They assumed the cracked plate domain to be an assemblage of small subdomains with the appropriate functions formed and led to a governing eigenvalue equation. Ramamurti and Neogy [21] have applied the generalized Rayleigh–Ritz method to determine the natural frequency of cracked cantilevered plates. The results show that the natural frequency decreases with increasing crack length at the same plate. All the above literature has made the conclusion that the natural frequency and the vibration amplitude are functions of the crack location and length.

In recent years, significant efforts have been published in the area of non-destructive damage evaluation for damage identification in structures [22–30]. These methods are based on the fact that local damages usually cause decrease in the structure stiffness, which produces the change in vibration characteristics (such as natural frequencies, mode shapes and curvature mode shapes) of the structure. When the changes of the vibration characteristics are examined, the location and magnitude of the structural damage can be identified. Among these vibration characteristics, natural frequency is one of the most common modal features are used in crack detection because it can be measured most conveniently and accurately.

However, natural frequency changes alone may not be sufficient for a unique identification of the structural damage. This is because cracks associated with different crack lengths but at two different dynamic conditions or with similar crack lengths but at two different locations may cause the same amount of frequency change. Dynamic characteristics are sensitive to change in boundary conditions, cracked locations, crack ratios, aspect ratios, amplitudes, applying loads and environment factors et al. Moreover, many of the proposed methods require either a theoretical model of damage or a set of sensitivity values to be computed before physical measurements. Therefore, for successful utilization of vibration data as an analytic tool for damage identification, it is also necessary to understand the effects of all possible damage events at various dynamic conditions on the structure.

In this study, solutions to dynamic instability of a cracked plate subjected to periodic in-plane force along two opposite edges are determined. The solution for the dynamic analog of the von Karman's plate theory satisfying all the cracked boundary conditions is obtained by expressing the displacement and the force functions as double series in terms of appropriate beam functions. Using the Galerkin's method, the governing equation is reduced to a time-dependent Mathieu equation. The dynamic instability characteristics are investigated by the incremental harmonic balance (IHB) method. The IHB method has been successfully applied to various types of non-linear dynamic problems and discussed in a number of papers: for example, Lau et al. [31], Pierre and Dowell [32] and Lau and Yuan [33]. The rectangular plates are considered with the aspect ratio and crack ratio on the interval $0.5 \leqslant a/2b \leqslant 2.0$ and $0.0 \leqslant c/a \leqslant 0.5$, respectively, and the amplitude of vibration of the plate is regarded as another parameter of the system. The effects of various parameters on the instability and the non-linear response characteristics of the plates have been provided in this study.

2. Governing equations

The equations of motion for generally isotropic plates based on von Karman's plate theory are given by Dym and Shames [34], and can be reduced to the following set of equations:

$$\nabla^4 F = -\frac{Eh}{2} L(w, w) \tag{1a}$$

$$D\nabla^4 w = L(F, w) - \rho h w_{tt} \tag{1b}$$

where w is the displacement at midsurface in z-direction of rectangular Cartesian coordinates; h is the thickness of the plate; t is the time, and ρ is the mass density per unit volume. The flexural rigidity is defined as $D = Eh^3/(12(1-v^2))$; E is Young's module and v is Poisson ratio. The stress function, F, is defined by

$$N_x = \frac{\partial^2 F}{\partial y^2}, \quad N_y = \frac{\partial^2 F}{\partial x^2}, \quad N_{xy} = -\frac{\partial^2 F}{\partial x\,\partial y} \tag{2}$$

where N_x, and N_y are the normal stresses in x-direction and y-direction, respectively, and N_{xy} is the shear stress. Further, the operator for the partial differentiation are defined as

$$\begin{aligned}
\nabla^4 F &= \frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2\,\partial y^3} + \frac{\partial^4 F}{\partial y^4} \\
\nabla^4 w &= \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^3} + \frac{\partial^4 w}{\partial y^4} \\
L(w, w) &= 2\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\left(\frac{\partial w}{\partial x\,\partial y}\right)^2 \\
L(F, w) &= \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{\partial x\,\partial y}\frac{\partial^2 w}{\partial x\,\partial y} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2}
\end{aligned} \tag{3}$$