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# A mathematical model for the behavior of laminated glass beams

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**Abstract**

Two-ply laminated architectural glass used in automotive industry since 1914, now become an important element in construction industry as well. Regarding its importance, a theoretical model is needed for the laminated glass beams investigated mostly experimentally so far. In this paper, a mathematical model for the behavior of laminated glass beams is introduced. The minimum total potential energy principle is employed in developing the mathematical model by assuming large deflection for a laminated composite beam since glass beams are very thin. The model is then validated by the experimental and finite element models for the simply and fixed supported beams respectively. It is presented that the behavior of laminated glass beams under large deflections could be linear or nonlinear regarding the boundary conditions or constraints.


*Keywords:* Laminated glass; PVB; Beam; Nonlinear

## 1. Introduction

Architectural laminated glass beam comprises two layers of glass with a layer of elastomeric polymer in between, which is usually called polyvinyl butyral (PVB). This composite unit is used for safety in glazing applications. When it is broken under wind load or for other reasons, the soft plastic interlayer between the glass layers keeps the unit intact and prevents dangerous shards from splitting and lacerating people.

The layered combination of very hard (glass) and very soft (interlayer) materials to form a laminated glass unit makes the unit behave in a very unusual manner due to the order difference in modulus of elasticity of materials. On the other hand, since the glasses are very thin the unit undergoes large deflections even under its own weight. Large deflection of a laminated glass beam could produces nonlinear behavior for the type of boundary conditions that create membrane forces, such as fully fixed end conditions. But contrary to the case with fully fixed end conditions, the fully simply supported laminated glass beam behaves linearly even at large deformations since axial constraints are not developing.

## 2. Previous research

Studies on the bending of simply supported laminated glass beams under distributed, three-point and four-point loadings have been conducted by Hooper [1], Behr et al. [2], Edel [3] and Norville et al. [4].

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**Nomenclature**

| | |
|---|---|
| $\underline{A}$ | coefficient matrix |
| $A_1$, $A_2$ | cross-sectional area of top and bottom glass ply |
| $b$ | width of beam |
| $E$ | modulus of elasticity of glass |
| $G$ | shear modulus of interlayer |
| $h$ | thickness of single glass ply |
| $h_t$ | total thickness of laminated glass unit |
| $h_1$, $h_2$ | thickness of top and bottom glass plies |
| $I$ | cross-sectional moment of inertia of the laminated glass section |
| $I_1$, $I_2$ | cross-sectional moment of inertia of the top and bottom glass plies, respectively |
| $L$ | half length of beam |
| $M$ | bending moment in the beam |
| $N_1$, $N_2$ | cross-sectional force in the top and bottom glass ply |
| *num* | number of discrete points along the beam |
| $P$ | point load applied at the middle of beam |
| $q$ | uniformlydistributed load applied over the length of beam |
| $\underline{R}$ | right-hand side vector |
| $t$ | thickness of the interlayer |
| $U$ | total strain energy |
| $U_m^i$ | membrane strain energy for the top and bottom ply |
| $U_b^i$ | bending strain energy for the top and bottom ply |
| $U_{\tau_{xz}}$ | shear strain energy for the interlayer |
| $u_1$, $u_2$ | in-plane displacement for the top and bottom ply in the $x$ direction |
| $V$ | potential energy of applied loads |
| $\underline{w}$ | lateral displacement vector |
| $w_{max}$ | maximum displacement in the plate |
| $wo(i)$ | lateral displacement calculated in the previous step |
| $x$, $z$ | rectangular coordinates |
| $\alpha$, $\beta$, $\lambda$ | parameters related with section geometry and material properties |
| $\varepsilon_x^i$ | axial strain energy for the top and bottom ply |
| $\varepsilon_b^i$ | bending strain energy for the top and bottom ply |
| $\gamma_{xz}$ | shear strain in the interlayer |
| $\eta$ | underrelaxation parameter for convergence |
| $\Pi$ | total potential energy of the system |
| $\sigma_1^{top}$, $\sigma_1^{bot}$ | bending stresses on the top and bottom surfaces of the top ply |
| $\sigma_2^{top}$, $\sigma_2^{bot}$ | bending stresses on the top and bottom surfaces of the bottom ply |
| $\tau$ | shear stress in the interlayer |

Simply supported beams considered in the above studies behave linearly with respect to load. The first study on laminated glass beams was conducted by Hooper [1]. He derived a mathematical model for the bending of laminated glass beams under four-point loading based on a solution to a problem of bending of parallel beams interconnected by cross members. In this approach, the solution method centered on replacing the discrete assemblage of interconnecting cross-members by a continuous medium of equivalent stiffness, the medium itself being firmly attached to the beams at each interface. Hooper saw that this latter condition corresponded precisely to that prevailing in the case of architectural laminated glass, in which a relatively soft continuous layer is confined between two glass layers, and remains in adhesive contact with them during bending. He solved the relevant differential equation in terms of the applied bending moment and the axial force in one of the plies via Laplace transform, to plot three influence factors $K_1$, $K_2$ and $K_3$, respectively, proportional to the axial force in one of the plies, shear strain in the interlayer and central deflection. He noted that since PVB is a viscoelastic material its shear modulus can be written as a function of time which asymptotically approaches zero as time increases. He also conducted tests on laminated glass beams with PVB interlayer under short (<3 min) and long (80 days) loading durations.

Because of difficulties in modeling of laminated glasses, researchers first try to model the laminated glasses as a layered glass unit formed by two glass layers put on top of each other with no interlayer and no friction in between, or monolithic glass having a thickness equal to the total thickness of glass layers in laminated glass unit. Behr et al. [5], considering that the shear modulus of the PVB interlayer with plastic and viscoelastic properties is several orders of magnitude less than that of glass, concluded that the effective shear modulus of the interlayer is proved to be very low. This implied that laminated glass units could be approximated as "layered" units with no shear connection between the adjacent glass plates, and finite difference solution could be used to calculate the limiting cases of layered and monolithic systems. They conducted a series of experiments on layered, monolithic and laminated glass units. It was observed that at room temperature the laminated glass unit deflected like a monolithic glass plate having the same glass thickness, whereas at higher temperatures it deflected increasingly like a layered glass unit having the same glass thickness. It was also observed that the