Available online at www.sciencedirect.com
SCIENCE DIRECT®
Computers and Structures 83 (2005) 1191–1204
Computers & Structures
www.elsevier.com/locate/compstruc

Computational aspects of structural shape control

Franz Ziegler *

Department of Civil Engineering, Vienna University of Technology, Wiedner-Hstr. 8-10/E201, A-1040 Vienna, Austria

Accepted 25 August 2004
Available online 3 March 2005

Abstract

The goal of shape control is nullification of the structural deformations caused by certain external disturbances, mainly body forces and surface traction. Dynamic structural shape control is concerned with vibration suppression. Determination of a proper distributed actuation is understood through the interaction of structural mechanics (of smart materials) and control engineering. Imposed strains (eigenstrains) are of quasistatic thermal nature in graded materials, or mainly make use of the piezoelectric effect in smart composites containing conventional ferroelectric polycrystals, natural crystals or special polymers. For large scale structures tendons or built-in hydraulic actuators are available. For discretized or discrete structures (e.g., trusses) the general solution is given in terms of the flexibility matrix and the two, orthogonal subspaces, of the impotent and nilpotent eigenstrains in Hilbert space are mentioned. Since impotent eigenstrains do not produce stress, they are ideally suited for shape control. Further, vibration suppression is discussed in the context of separation in space and time of the forcing function. In those cases, knowledge of the quasistatic load deformation suffices to define the distributed actuators producing impotent eigenstrain.

Keywords: Static and dynamic shape control; Thermal strain; Piezoelectric strain; Impotent eigenstrain; Nilpotent eigenstrain; Stiffness and flexibility matrix; Stress redistribution

1. Introduction

Haftka and Adelman [1] presented an analytical procedure for computing the temperature field in the supporting structure to minimize deviations of large space structures from their original shape calling the procedure "shape control of structures". Their detailed computational efforts considered a 55 m radiometer antenna supported by tetrahedral truss modules. One of the first attempts to apply shape control in Aeronautics in the open literature by Austin et al. [2] considered an adaptive wing of a fighter plane. An early application to rotary wings to suppress their vibrations is given by Nitzsche and Breitbach [3]. With the technology of smart materials developing at a vast stage, aerospace applications changed from futuristic aspects into applicability, and reviewing papers summarizing the state-of-the-art appeared, Crawley [4]. Further reviews, with emphasis on piezoelectricity and its application in disturbance sensing and control of flexible structures by Rao and Sunar [5], including active vibration control of laminated piezoelectric beams, plates, and shells by Saravanos and Heylinger [6], with emphasis on control, sensors and actuators by both, Tzou [7] and Lee [8] followed. A most recent review of static and dynamic shape

* Tel.: +43 1 58801 201 10; fax: +43 1 58801 201 99.
E-mail address: franz.ziegler@tuwien.ac.at

0045-7949/$ - see front matter
doi:10.1016/j.compstruc.2004.08.026

control by piezoelectric actuation is presented by Irschik [9]. Smart composite structures are considered by Tauchert et al. [10] and the overall developments including computational aspects are documented in the Proceedings edited by Gabbert [11], Bahei-el-Din and Dvorak [12], Gabbert and Tzou [13] and Watanabe and Ziegler [14].

With respect to shape control of smart structures, this problem has been tackled in general by non-linear optimization techniques, where a finite number of (shaped) actuator patches has been applied to the structure. The shaping problem is tackled by Pichler [15]. For an intelligent plate see Agrawal et al. [16], Varadan et al. [17], for composite plates and shells, see Koconis et al. [18]. Since a finite number of actuator patches is considered, or localized temperature fields are applied in graded materials, such a collocation produces the desired deflection of a flexible distributed-parameter system only approximately.

In the present paper, the direct solution of the quasistatic shape control problem in the context of the multiple field approach (i.e. considering the given structure in the background) is discussed, for details see Irschik and Ziegler [19]. An outlook on stress control is given by Nyashin et al. [20] and Nyashin and Lokhov [21]. The extension to dynamic problems is derived by Irschik and Pichler [22]. In the latter case, under conditions of time and space separable loads it is sufficient to control the much simpler quasistatic force deformation, see also Irschik et al. [23]. Emphasis is led upon the definition of proper eigenstrain distributions, for precise definitions see Reißner [24], however under the assumptions of unlimited intensity of the sources and their continuous distribution. In this sense, benchmark solutions are derived and their practical application becomes necessarily approximate. "Eigenstrain" is a generic name originally given by Mura [25], to inelastic strains resulting from thermal expansion, phase transformation, initial strains, plastic strains, and misfit strains. Other imposed strains produced e.g., by electrical fields in piezoelectric materials, are of the same nature and in this context understood as eigenstrains. Shape control is ideally performed by impotent eigenstrains which produce deformation but no stress. The separated stress control procedure, however, is subjected to severe constraints given by the local conditions of equilibrium, and is achieved by nilpotent eigenstrains. The latter produce stress but no deformations and thus have no influence on shape control. Classical examples are known in thermoelasticity, Ziegler and Irschik [26], or have been encountered in the case of flexural vibrations of piezoelectric beams and attributed to an electric field without deflection, Irschik et al. [27]. When properly defined, they can be used to redistribute the load stresses without influencing shape control.

2. (Quasi-)static shape control

The control of the deformations of linear elastic structures, produced by force loading, body forces and/or surface traction, by means of imposed eigenstrains is discussed in general terms of the associated boundary value problems. In this context, the Green's stress dyadic is assumed to be known in principle, and the class of impotent eigenstrains is determined by inspection and identified as compatible strains, i.e. they are equal to the strains produced by the given load, or alternatively, in case of a desirable deformation, the latter must be related to a distribution of a fictitious force load. This class of eigenstrains when activated by distributed "actuators", produce deformations but no stresses in the structure under consideration, say in the background. Interaction of the force strain field and the eigenstrains, properly produced by these actuators, interact in the background in the sense of the multiple field concept.

2.1. Force load

We consider the general boundary value problem of a linear elastic, possibly anisotropic body loaded by body forces and/or surface traction. The local equilibrium requires

$$\operatorname{div} \underline{\sigma}_{(\mathrm{F})} + \underline{b} = \underline{0} \tag{1}$$

where in this section the stress tensor $\underline{\sigma} = \underline{\sigma}_{(\mathrm{F})}$ is substituted. On part of the boundary, kinematic boundary conditions apply

$$\Gamma_u : \underline{u} = \underline{0}, \ldots \tag{2a}$$

where the displacement produced by the force loading $\underline{u} = \underline{u}_{(\mathrm{F})}$ is understood in Eq. (2a). On the remaining part, the traction are prescribed

$$\Gamma_\sigma : \underline{\sigma} \cdot \underline{n} = \underline{t}^{(n)} \tag{2b}$$

where $\underline{\sigma} = \underline{\sigma}_{(\mathrm{F})}$ in Eq. (2b). Within the validity of linearized geometric relations for the strain produced by the force load,

$$\varepsilon_{ij(\mathrm{F})} = \tfrac{1}{2}\left(u_{i,j} + u_{j,i}\right)_{(\mathrm{F})} \tag{3}$$

and with Hooke's law taken into account

$$\varepsilon_{ij(\mathrm{F})} = C_{ijlm}\sigma_{lm(\mathrm{F})} \tag{4}$$

the solution of the force–displacements $u_{i(\mathrm{F})}$, $i = 1,2,3$, by means of the principle of virtual forces, see Ziegler [27], is given in the form of the volume integral, a complementary Green's formula,

$$1 \cdot u_{k(\mathrm{F})}(\underline{x}) = \int_V \tilde{\sigma}_{ij(k)}\left(\underline{\xi}, \underline{x}\right)\varepsilon_{ij(\mathrm{F})}\left(\underline{\xi}\right) \mathrm{d}V\left(\underline{\xi}\right) \tag{5}$$

which represents a virtual work relation. The Green's stress dyadic $\tilde{\sigma}_{ij(k)}$ produced by a unit single force