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Nonlinear vibration of finitely-electroconductive plate strips in an axial magnetic field

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Abstract

The vibrational behavior of geometrically nonlinear, finitely electroconductive, isotropic elastic plate strips immersed in an axial magnetic field is investigated. Kirchhoff hypothesis in conjunction with von-Kármán's concept of strain is used to model the mechanical part, while the assumptions proposed by Ambartsumyan et al. are adopted to model the distribution of electric and magnetic disturbances through the plate-strip thickness. A system of nonlinear singular integro-differential equations are obtained, and by applying the Galerkin's method, a third order nonlinear ordinary differential equation is derived. The influence of the magnetic field and electroconductivity on the plate-strip vibration is investigated and analytical solutions of the nonlinear fundamental frequency are obtained via the Method of Multiple Scales for two special cases, consisting of weak magnetic field and high electroconductivity. Finally, some pertinent conclusions are provided.

Keywords: Plate vibration; Finite electroconductivity; Magnetoelastic; Geometric nonlinearities

1. Introduction

One of the widely adopted assumptions to deal with electroconductive elastic structures immersed in a magnetic field is that of the infinite electric conductivity. However, due to the extremely large variations of electromagnetic properties of the materials, the universal adoption of such an assumption becomes really questionable. In fact, one of the reasons of its adoption is due to the considerable simplification of the governing equations, which otherwise are very difficult to handle. However, the finiteness of electroconductivity does produce new phenomena, as evidenced by some incipient available results obtained within the linearized framework of the problem. The research work reported in the present article was prompted by the requirement of putting into evidence its implications. Due to the high intricacy of the problem being addressed, this study will be confined to the nonlinear vibration of finitely electroconductive plate strips immersed in an axial, static magnetic field. The objective herein is to investigate, in the presence of a magnetic field, the influence of finiteness of electroconductivity on the nonlinear vibration frequencies and amplitude of the plate strips.

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Nomenclature

c	speed of light, 3×10^{10}cm/s	σ	electroconductivity of the plate-strip
D	flexural stiffness, $2E\,h^3/[3(1-v^2)]$	ρ_0	mass density (per volume) of the plate strip
E	Young's modulus of the plate strip	τ	non-dimensional time variable, $\omega_0 t$
g	non-dimensional electroconductivity parameter, see Eq. (30a)	χ	x_3 component of the perturbed magnetic field, see Eq. (9)
$2h$	thickness of the plate strip	ψ	x_2 component of the induced electric field, see Eq. (9)
H_{01}	x_1 component of the unperturbed magnetic field	ω_0	reference frequency, $(4\pi^2/\ell_1^2)(Eh^2/[3(1-v^2)\rho_0])^{\frac{1}{2}}$
$2\ell_1$	width of the plate strip	Φ	potential of the perturbed magnetic field, see Eq. (11)
r	non-dimensional magnetic field intensity parameter, see Eq. (30b)		
v	Poisson's ratio	Ω	non-dimensional frequency, ω/ω_0

2. Field equations

The system of equations that govern the magnetoelastic vibrations of thin plate-strips will be rigorously derived from the 3D theory of electromagnetoelastic continuum (see e.g., [1–3]). In order to be reasonably self-contained, in what follows, the equations of electrodynamics and the equations of motion of 3D elastic media will be summarized.

Expressed in *Gauss*' system of units, in the absence of electrical free charges, the relevant field equations are (see e.g., [1–8]):

Faraday's Law : $$\operatorname{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \quad (1)$$

Ampère's Law : $$\operatorname{curl}\mathbf{H} = \frac{4\pi}{c}\mathbf{J} \quad (2)$$

Gauss' Law : $$\operatorname{div}\mathbf{D} = 0 \quad (3)$$

Conservation of flux : $$\operatorname{div}\mathbf{B} = 0 \quad (4)$$

Equations of motion : $$\left[S_{jr}(\delta_{ir} + V_{i,r})\right]_{,j} + f_i = \rho_0 \frac{\partial^2 V_i}{\partial t^2} \quad (5)$$

In Eqs. (1) and (2), c is the speed of light($c = 3 \times 10^{10}$cm/sec), while in Eqs. (5), δ_{ij} is the Kronecker delta, S_{ij} ($\equiv S_{ji}$) are the components of the second Piola–Kirchhoff stress tensor, $\mathbf{V}$ is the displacement vector of the 3-D material points with components V_i ($i = 1,2,3$), and f_i are the components of the ponderomotive force vector $\mathbf{f}$ per unit volume, which can be expressed as:

$$\mathbf{f} = \frac{1}{c}(\mathbf{J} \times \mathbf{B}) \quad (6)$$

For isotropic, linear and elastic non-ferromagnetic materials, the constitutive equations are:

$$\mathbf{B} = \mathbf{H}, \qquad \mathbf{D} = \mathbf{E}, \qquad \mathbf{J} = \sigma\left(\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{V}}{\partial t} \times \mathbf{B}\right) \quad (7a)$$

$$S_{ij} = \frac{E}{(1+v)}\left[\frac{v}{(1-2v)}e_{kk}\delta_{ij} + e_{ij}\right], \quad (i,j) = \overline{1,\ 3} \quad (7b)$$

where σ is the electric conductivity, E and v are the Young's modulus and Poisson's ratio of the material, respectively, while e_{ij} are the components of the Lagrangian strain tensor. The associated boundary conditions are (see, e.g., [1–8])

$$\mathbf{n} \times (\mathbf{E} - \mathbf{E}_e) = 0, \quad \mathbf{n} \times (\mathbf{H} - \mathbf{H}_e) = \mathbf{J}_s, \quad \mathbf{n} \cdot (\mathbf{B} - \mathbf{B}_e) = 0 \quad (8a)$$

$$n_i(S_{ij} - S_{jr}V_{i,r} + \mathbb{T}_{ij}) = F_j + n_i(\mathbb{T}_{ij})_e \quad (8b)$$

In Eqs. (8a) and (8b), $\mathbf{n}$ is the unit vector on the external normal of the deformed surface of the plate strip; $\mathbf{J}_s$ is the electric current vector on the surface; F_j are the components of the surface load vector $\mathbf{F}$ of mechanical origin; the subscript "e" identifies the quantities associated with the domain outside of the plate strip (i.e. the vacuum); $\mathbb{T}_{ij} = 1/(4\pi)[B_iH_j - (1/2)\mathbf{H} \cdot \mathbf{H}\delta_{ij}]$ is the Maxwell stress tensor; while $(\mathbb{T}_{ij})_e$, $\mathbf{E}_e$, $\mathbf{H}_e$ and $\mathbf{B}_e$ are the Maxwell stress tensor, electric field and magnetic induction vectors in the outer space, respectively.

3. Formulation of the governing system

The geometrical characteristics of the thin plate strip to be investigated is shown in Fig. 1. The material properties are assumed to be homogeneous and isotropic. The reduction of the 3-D equations to the 1-D counterparts governing the motion the plate strip is achieved via