Available online at www.sciencedirect.com

SCIENCE DIRECT®

Computers and Structures 83 (2005) 1251–1265

Computers & Structures

www.elsevier.com/locate/compstruc

Fracture analysis of piezoelectric materials with an arbitrarily oriented crack using energy density theory

Ching-Hwei Chue *, Shih-Ming Weng

Department of Mechanical Engineering, National Cheng Kung University, No. 1 Da-Shiue Road, Tainan, Taiwan, ROC

Accepted 29 September 2004
Available online 2 March 2005

Abstract

The fracture behavior of an arbitrarily oriented crack in a piezoelectric medium is studied in this paper by using the energy density theory. The governing parameters used in the fracture analysis are the remote mechanical load σ^{∞}, remote electric field E^{∞}, and the polarization orientation η with respect to the crack plane. The solution for the energy density factor S is modified to include the stress intensity factors K_{I} and K_{II}, the electric displacement intensity factor K_{D} (or the electric field intensity factor K_{E}) and the polarization orientation η. This paper's theoretical results discuss the importance of the polarization orientation on the crack driving force $S_{\min}$ and the crack propagating angle θ_0. For an arbitrarily oriented crack, a negative electric field was found to impede crack growth whereas a positive electric field enhanced it. The driving force and the crack propagation angle were also found to significantly influence polarization η. The ratio R (=$E^{\infty}/\sigma^{\infty}$) was used as a parameter to judge the significance of mechanical load or electric field. The inadequacy of using the energy release rate in predicting the fracture behavior is also discussed.

Keywords: Piezoelectric materials; Fracture mechanics; Energy density theory; Arbitrarily oriented poling

1. Introduction

Due to the electromechanical coupling behavior, piezoelectric ceramics (such as PZT4, PZT-5, etc.) are widely used in smart structures or electronic devices. Both mechanical and electric loads are applied on these devices and cause highly concentrated stresses. Since the materials are very brittle, the theoretically singular stresses can lead to crack extension starting at various discontinuities at the internal electrode edge (Furuta and Uchino [1]). In order to improve the performance of these devices, the fracture phenomenon of piezoelectric materials has to be better understood. The fracture behavior of piezoelectric material has been widely discussed in the recent years. A major limitation of these studies is the assumption that the boundary condition of crack surface is permeable or impermeable, i.e., the electric displacement vanishes or continues inside the crack for impermeable or permeable crack, respectively. Parton [2] assumed a slit crack in a piezoelectric solid is traction free as well as the electric potential and normal

* Corresponding author. Tel.: +886 6 2757575x62165; fax: +886 6 2363950.
E-mail address: chchue@mail.ncku.edu.tw (C.-H. Chue).

0045-7949/$ - see front matter
doi:10.1016/j.compstruc.2004.09.016

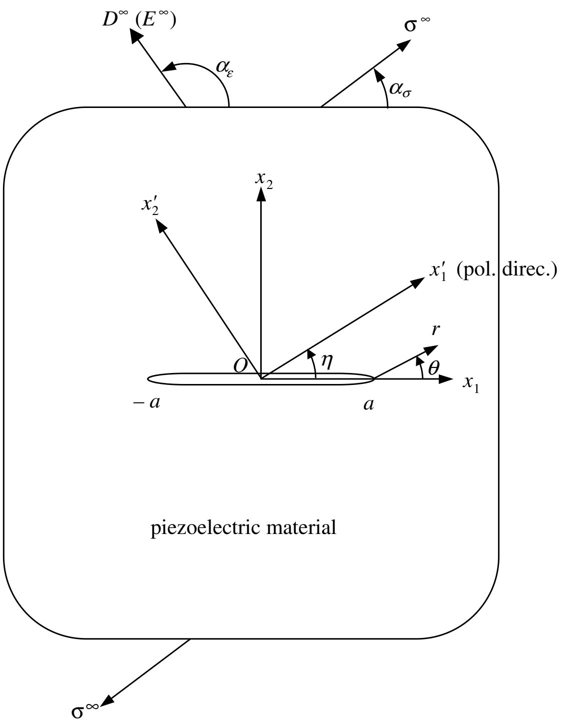

Fig. 1. Schematic diagram of a center crack with arbitrary poling direction.

component of the electric displacement being continuous across the crack surface. This boundary condition is referred to as a true piezoelectric permeable problem. However, Deeg [3] stated that since the dielectric constant of air or vacuum inside the crack is much smaller than the dielectric permittivity of the material, the crack must be treated as impermeable problem.

It is known that the three-dimensional piezoelectric problems can be decoupled into inplane and antiplane problems based on the poling direction of the materials. If the material is poled in $x_1 - x_2$ plane (Fig. 1), it results in a inplane electro-elastic field which involves stresses $(\sigma_x, \sigma_y, \tau_{xy})$, displacements (u, v), electrical displacements (D_x, D_y), and electric fields (E_x, E_y). Many researchers have addressed the crack problems subjected to inplane electrical and mechanical loads [4–10].

Due to the coupling between mechanical and electrical effects, the prediction of the fracture behavior of a crack in a piezoelectric medium would seem to be more complicated than that in an elastic material. Energy release rate concept has been widely used as a driving force for a cracked piezoelectric body [11–15]. However, this theory cannot differentiate between a positive and negative electric field. In Mode I and mixed mode fracture experiments, Park and Sun [16] have shown that positive or negative electric field can enhance or impede crack propagation, respectively. They proposed using the mechanical part of the total energy release rate to explain results. However, from a physics point of view, it is inconsistent to separate the total energy release rate into electrical and mechanical parts [16]. In the last few years, the energy density theory has been applied again to predict the piezoelectric crack problems [17–19]. Some researchers [20,21] have observed experimental results that differ with Park and Sun [16]. The diverse range of experimental results emphasizes the complex fracture behavior of piezoelectric material.

Except for Ref. [15], all of the researchers have assumed that the piezoelectric material is poled along or normal to the crack surface. Xu and Rajapakse [15] have addressed the role of the poling direction played in the fracture prediction by obtaining the electric–elastic field of an arbitrarily oriented void/crack. They used the maximum circumference stress theory [22] to predict the crack propagation direction. They also used the crack opening displacements (COD) criterion to explain the fact that a positive or negative electric field enhances or impedes crack propagation. Xu and Rajapakse [23] considered an arbitrarily oriented branched crack in a two-dimensional piezoelectric solid. The electroelastic fields at a branch tip have complex dependence on branch length, branch angle, crack orientation and the type of loading. By taking fracture toughness anisotropy into consideration, the criterion of modified hoop stress intensity is applied to predict potential branching directions.

In this paper, we adopt the same formulation used in Ref. [15] to get the expression of energy density factor (S) for impermeable crack when the piezoelectric material is poled in an arbitrary direction. The effects of the poling direction on the crack propagation angle and the driving force ($S_{\min}$) are examined for positive or negative electric field.

2. Basic equations

Consider a center crack in an arbitrarily poled piezoelectric plane (Fig. 1). The $x_1 - x_2$ plane is set up such that the x_1-axis and x_2-axis are parallel and normal to the crack surface, respectively. The material is poled in a direction, which makes an angle η with x_1-axis. A remote uniform stress (σ^{∞}) and electric displacement (D^{∞}) are applied with orientation angles of α_{σ} and α_{ε} as shown in Fig. 1. The constitutive equation of piezoelectric material polarized along x_1'-axis can be expressed as: