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Anisotropic and piezoelectric materials fracture analysis by BEM

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Abstract

A mixed boundary element approach for two-dimensional anisotropic and piezoelectric fracture mechanics problems is presented in this paper. The numerical approach is based on displacement and traction integral equations for external and crack boundaries, respectively. Integrals with strongly singular and hypersingular kernels are analytically transformed into weakly singular and regular integrals prior to any numerical evaluation. This is achieved by the simple election of an integration variable consistent with the material characteristic parameters. The generality of the method allows for the use of curved and quarter-point elements and the evaluation of stress and electric displacement intensity factors from nodal values next to the crack tip. Several crack problems in anisotropic and piezoelectric materials are solved. The obtained results are in good agreement with previous solutions for cases where these solutions exist. The present BE approach is more general and simple than previous procedures and has allowed for solution of a variety of crack problems including curved cracks in piezoelectric materials which had never been studied before.

Keywords: Boundary element method; Piezoelectric solids; Anisotropic solids; Fracture mechanics; Curved crack; Quarter-point element

1. Introduction

Piezoelectric behavior was identified by Pierre and Jacques Curie in 1880. They observed that some materials produce an electric field when deformed and undergo deformation when subjected to an electrical field. Since the mid of the twentieth century, there have been piezoelectric ceramics with a piezoelectric ratio between electric field and mechanical stress (and between mechanical strain and electric displacement), two orders of magnitude higher than that of natural piezoelectric materials. Lead zirconate titanate (PZT) is the most widely used piezoceramic and polyvinylidene fluoride (PVDF) the most extended piezopolymer. They were first produced in 1946 and 1969, respectively. Piezoelectric materials have become the base for construction of sensors, transducers, actuators and adaptive structures. Modeling of piezoelectrics is complicated by the fact that they exhibit not only electro-elastic coupling but anisotropic

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behavior as well. Piezoelectric effect can only appear in crystals that lack a center of symmetry; therefore, they are always anisotropic.

Piezoelectric materials are brittle and due to manufacturing process and complex electromechanical loads they are likely to develop cracks. The understanding and evaluation of the fracture process in piezoelectric materials is crucial to the advancement of modern intelligent material systems. Among the most significant publications on the field of piezoelectric materials fracture mechanics, one can cite the works of Barnett and Lothe [1], Deeg [2], Pak [3], Suo et al. [4], Sosa [5], Park and Sun [6] and Wu et al. [7].

It is well known that the boundary element method (BEM) presents significant advantages over other numerical techniques for the analysis of fracture mechanics problems. This fact has led to the publication of several BE approaches for the analysis of cracks in piezoelectric solids in the last few years. The main difficulties in the field are related to derivation and integration of fundamental solutions for two- and three-dimensional static and dynamic problems. Pan [8] presented a single domain BE formulation for 2-D static crack problems. He derived the fundamental solution using the complex variable function method [9–11] and computed the hypersingular integrals using a numerical quadrature. Liu and Fan [12] established some of the basic equations in a rigorous way and addressed the question of degeneration for cracks and thin shell-like problems. Rajapakse and Xu [13] and Xu and Rajapakse [14], used Lekhnitskii's formalism and distributed dislocation modeling to derive special Green's functions for an infinite medium containing a crack. They studied different crack geometries including branched cracks. Denda and Lua [15] developed a BEM formulation using Stroh's formalism to derive the fundamental solution but they did not show any numerical result. Davi and Milazo [16] presented a multidomain approach based on the conventional BE formulation. The fundamental solution was obtained by a variant of Lekhnitskii's functions method similar to that used by Rajapakse and Xu [13,14]. All these papers deal with 2-D domains.

Very few papers on BE formulations for three-dimensional piezoelectric solids have been published. Particular attention should be paid to the paper by Hill and Farris [17] where the fundamental solution is obtained using the Radon transform as in Deeg's work [2] and some simple numerical examples are analyzed using quadratic elements. A 3-D fundamental solution for displacement and electric potential discontinuity was presented by Zhao et al. [18] and was applied to the case of a circular crack in infinite domain [19].

Boundary elements for time harmonic 2-D piezoelectric problems have been developed and applied to natural frequencies computation by Denda and Araki [20]. Kogl and Gaul [21] studied 3-D dynamic problems using the static fundamental solution as derived by Deeg [2] and a Dual Reciprocity formulation. Daros and Antes [22] did a pioneering work obtaining a fundamental solution for 3-D dynamic piezoelectric problems.

Sollero and Aliabadi [23] and Pan and coworkers [24–26] studied anisotropic 2-D crack problems using the mixed formulation (dual). Pan [8] extended his previous formulation to piezoelectric solids. Those two-dimensional approaches for anisotropic and piezoelectric crack problems contain geometrical limitations (on the first case) or are based on numerical integration procedures which are less accurate and less robust than the simple general analytical approach presented in this paper.

In the first part of this paper, a general mixed BE formulation based on displacement and traction integral equations for 2-D cracked anisotropic media is presented. Next, the formulation is extended to piezoelectric solids. In both cases, a new regularization procedure based on a simple change of variables is presented. All the strongly singular or hypersingular integrals are transformed into regular integrals and simple singular integrals with known analytical solution. The traction (and normal electric displacement) integral representation is written for the crack surface and the displacement (and electric potential) integral representation for the external boundaries. The basic variables are the opening displacement (and the electric potential discontinuity) along the crack, and the displacement (and electric potential) and traction (and normal electric displacement), on the external boundaries. Quadratic quarter-point elements are used to represent the crack opening displacement (COD) and potential discontinuity near the crack tip. Standard curved or straight quadratic elements are used for the rest of the crack and the external boundaries. Stress Intensity Factors (SIF) are computed in a direct way from the COD and the electric potential discontinuity at a point extremely close to the tip. Numerical examples regarding full plane, half plane and bounded domain crack problems are analyzed. The obtained results are compared with those existing in the literature. The present BE procedure is shown to be accurate and robust.

2. Basic equations

2.1. Elastic anisotropic materials

The mixed formulation for the BE solution of crack problems is based on both displacement and traction integral representations. In the case of zero body forces, the 2-D displacement integral representation for a point with coordinates ξ_1 and ξ_2 in an elastic anisotropic solid can be written as