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Numerical simulations of free-interface fluids by a multi-integrated moment method

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Abstract

This paper presents a work on constructing numerical model for multi-fluid dynamics by utilizing the integrated moments of the dependent variables. The presented method, namely VSIAM3 (Volume/Surface Integrated Average based Multi-Moment Method), employs two integrated moments which are called volume integrated average (VIA) and SIA (Surface Integrated Average), and results in a new finite volume formulation for solving general fluid dynamical problems. Using both VIA and SIA as the model variables, VSIAM3 has compact mesh stencil and larger flexibility in designing the computational algorithm of the solution procedure, thus is suited for dealing with complicated flows. Numerical aspects which are under the framework of VSIAM3 to accommodate the obstacle computation and the interface capturing will be discussed with numerical examples.

Keywords: Computational fluid dynamics; CIP; Finite volume method; Moving interface; Complex geometry; Multi-fluid flow

1. Introduction

Finite volume method (FVM), which is formulated directly from the conservation laws for continuum dynamics in their integral form, provides a general framework for fluid dynamics and possesses many attractive properties, such as conservation and flexibility in treating irregular geometry. An FVM is usually cast in a formulation which predicts the integrated average value for each mesh cell by examining the net flux across the cell boundary. The numerical flux on cell boundary in a conventional FVM is evaluated by a consistent approximation based on the cell-integrated average value. The cell-integrated average (or the volume integrated average as we use hereafter in this paper) in a conventional FVM is usually the only quantity memorized and predicted in the numerical model.

Being an alternative to the conventional FVM for numerical advection , CIP-CSL (Constrained Interpolation Profile-Conservative Semi-Lagrangian) schemes [25,21,22] include the cell boundary values as other model variables in addition to the cell-integrated average. Being a model variable, the cell boundary value is memorized and updated at each step rather than interpolated from the cell-integrated averages. The cell integrated average in 3D is the volume integrated average

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Nomenclature

t	time
x, y, z	three independent spatial variables in Cartesian coordinate
ϕ	any scalar quantity
$\mathbf{u}$	velocity vector with the three components in Cartesian coordinate denoted by u, v and w
p	pressure
ρ	density
$\mathbf{F}$	vector of body force with the three components in Cartesian coordinate denoted by F_x, F_y and F_z.
τ	tensor of viscous stress
$\mathbf{n}$	outgoing normal vector for the control volume
V	control volume of computational mesh (volume element)
S^x	surface element normal to x axis
S^y	surface element normal to y axis
S^z	surface element normal to z axis
ω	fluid fraction for volume element
γ	fluid fraction for surface element

Operators

∇	gradient
$\mathscr{D}$	divergence
$\mathscr{F}$	advection flux with the three components denoted by $\mathscr{F}_x$, $\mathscr{F}_y$ and $\mathscr{F}_z$
$\overline{{}^{V}(\ )}$	volume integrated average over V
$\overline{S_x(\ )}$	surface integrated average over S_x
$\overline{S_y(\ )}$	surface integrated average over S_y
$\overline{S_z(\ )}$	surface integrated average over S_z
$\overline{{}^{x}(\ )}$	integrated average over a line segment in x direction
$\overline{{}^{0}(\ )}$	point value
$\delta_x(\ )$	difference in respect to x direction
$\delta_y(\ )$	difference in respect to y direction
$\delta_z(\ )$	difference in respect to z direction
$\overline{(\ )}^{x}$	average in respect to x direction
$\overline{(\ )}^{y}$	average in respect to y direction
$\overline{(\ )}^{z}$	average in respect to z direction

Subscripts

ijk	index for volume element
$i+\frac{1}{2}jk$	index for surface element normal to x direction
$ij+\frac{1}{2}k$	index for surface element normal to y direction
$ijk+\frac{1}{2}$	index for surface element normal to z direction

Superscripts

n	denotation for the nth step
$\diamond$ or $*$	provisional value between substeps
$\bar{\diamond}$	trajectory-averaged value

(VIA), and the cell boundary value is correspondingly interpreted as the surface integrated average (SIA). The concept of using more than one "moments" for each physical variable comes from the CIP (Cubic-Interpolated Pseudo-particle or Constrained Interpolation Profile) scheme [23,24,26], where the use is made of the first order derivatives of the time-evolution variable as the extra "moments" which are then predicted in the numerical model as other variables.

We should make a few remarks concerning the general term "moment" used in this paper. We use the term "moment" herein following the definitions in [12], where the cell average, slope and curvature of a distribution are referred to zero-order, first-order and second-order moments of the approximation function respectively. Rigorously speaking, the meaning of the word "moment" here is slightly different from that in Prather and other schemes in the sense that a moment usually appears as the coefficient of the basis function used in the latter. Nevertheless, we preferably use "moment" as a general term to refer to various quantities measuring the spatial distribution of a given field. "Multi-Moment" reflects the characteristic of the CIP type schemes that store and carry forward more than one quantities (e.g. the grid-point value and the first-order derivative in the original CIP scheme) for a physical variable. In fact, we use in the present study two zero-order moments defined with respect to different dimensionality, i.e. the volume integrated average and the surface integrated average.

Retaining VIA and SIA to be the dependent variables, we have recently developed a method called VSIAM3 (Volume/Surface Integrated Average based Multi-Moment Method) [17,19] for general dynamical problems defined in time-space domain. The integrated moments which are hierarchically defined in different dimensionality are reciprocal in constructing numerical model. By using the spatially integrated moments, the VSIAM3 can naturally satisfy the conservation constraint imposed on the original dynamic system. With more than one moments in its formulation, VSIAM3 is different from the conventional FVM, and can be interpreted as a CIP finite volume formulation.

The VSIAM3 possesses some numerical features which are well suited for handling complex geometry