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Review

A posteriori error estimation techniques in practical finite element analysis

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Abstract

In this paper we review the basic concepts to obtain a posteriori error estimates for the finite element solution of an elliptic linear model problem. We give the basic ideas to establish global error estimates for the energy norm as well as goal-oriented error estimates. While we show how these error estimation techniques are employed for our simple model problem, the emphasis of the paper is on assessing whether these procedures are ready for use in practical linear finite element analysis. We conclude that the actually practical error estimation techniques do not provide mathematically proven bounds on the error and need to be used with care. The more accurate estimation procedures also do not provide proven bounds that, in general, can be computed efficiently. We also briefly comment upon the state of error estimations in nonlinear and transient analyses and when mixed methods are used.

Keywords: Finite element analysis; A posteriori error estimation; Goal-oriented error estimation; Dual problem; Practical procedures

Contents

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1. Introduction

The modeling of physical phenomena arising in engineering and the sciences leads to partial differential equations in space and time, expressing the mathematical model of the problem to be solved. In general, analytical solutions of these equations do not exist, hence numerical methods such as the finite element method are employed. A major feature of numerical methods is that they involve different sources of numerical errors [1,2]. The focus of this paper is only on the discretization error which is due to the finite element (polynomial) approximation of the solution. Hence, we assume that an appropriate mathematical model has been chosen and, even for this case, we are only concerned with one specific error, namely the discretization error arising in the finite element solution of this model.

Since the late 1970s several strategies have been developed to estimate the discretization errors of finite element solutions. Basically, there are two types of error estimation procedures available. So called a priori error estimators provide information on the asymptotic behavior of the discretization errors but are not designed to give an actual error estimate for a given mesh. In contrast, a posteriori error estimators employ the finite element solution itself to derive estimates of the actual solution errors. They are also used to steer adaptive schemes where either the mesh is locally refined (*h-version*) or the polynomial degree is raised (*p*-method). Most a posteriori error estimators developed prior to the mid-1990s focused on the global error in the energy norm. Then recently the theory was extended to estimate the error in particular quantities of interest. To understand the importance of this extension it must be realized that many local or global quantities of interest—such as deformations, stresses, drag and lift coefficients or the heat transfer of a structural part—can be obtained by applying a linear functional to the solution.