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Incompressible hydroelastic vibrations: finite element modelling of the elastogravity operator

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Abstract

This paper deals with the low frequency vibratory analysis of fluid–structure interactions in an elastic tank partially filled with an incompressible inviscid liquid. The originality of this work is to give an exact expression of the gravity interface operator whereas other standard hydroelastic formulations treat this effect through approximations. The properties of this so-called elastogravity operator will be studied here from theoretical and numerical point of view. Experimental measurements will validate the computational model and allow to quantify the effect of coupling between the liquid sloshing and the hydroelastic deformations of the structure.

Keywords: Fluid–structure interactions; Internal liquid; Sloshing; Hydroelasticity; Gravity; Hydrostatic pressure

1. Introduction

Many structures designed by mechanical engineers are intended to contain a more or less important quantity of liquid. Let us cite for example all the means of transport whose tanks are filled with liquid propellant (car, aircraft, space launcher, etc.), sea and road tankers, nuclear reactors or chemical industry containers. Many industrial domains are then interested in the problematic of the coupling between an elastic structure and an internal incompressible fluid, and a huge theoretical and numerical work has been done on this topic for the last decades, in particular in the aerospace domain (see for instance [1]).

We are interested here especially in the vibratory analysis of this coupled fluid–structure system and neglect all flow-induced vibration effects (see for instance [2] and also [3–5] for coupling with nonlinear structural displacements). To compute the eigenmodes of an elastic structure coupled with an internal inviscid and incompressible fluid, the standard hydroelastic model is generally used. However, this modelling assumes that the sloshing potential energy of the fluid, which is its gravity potential energy, can be neglected with respect to the deformation potential energy of the structure. Consequently, the gravity is completely omitted in this modelling. The fluid is then only represented by a kinetic energy and its contribution to the system is reduced to an added-mass effect [6]. The results obtained with this formulation are correct if the frequency domain of the coupled system eigenmodes is much higher than the frequency of the first sloshing eigenmodes. In the case of

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slender and flexible structures, such as space launchers or civil aircraft wings, the first structural eigenfrequencies are very low (between 1 and 2 Hz) and the decoupling between the liquid sloshing in tanks and the structural deformations is no longer a valid assumption. Some authors proposed to solve this problem by simply adding the gravity potential energy of the fluid to the Lagrangian of the system (see for instance [7,6]). In 1966, Tong had proposed a simplified expression for the fluid added-stiffness matrix due to gravity [8] which has been often used (see for instance [9]). However, in 1992, Morand and Ohayon proposed the first exact formulation of this operator [10] and the present study comes within the following of their works.

In this paper, we will first detail the derivation of the linear local equations of the problem when the effects of gravity are taken into account. The case of pressurized closed containers will also be considered. A symmetric variational formulation will then be deduced. The expression of the so-called *elastogravity operator* will be given and its properties will be discussed. The finite element discretization of this hydroelastic modelling with gravity will be carried out and a first experimental validation will be finally exposed.

2. Linearized local equations of the coupled system

Fig. 1 represents the generic internal fluid–structure system we consider here. A structure, denoted by Ω^S is partially filled by a fluid Ω^F. The fluid free surface and fluid–structure interface are respectively denoted by Γ and Σ_i. The tank ullage, which may be occupied by a pressurized gas, is denoted by Ω^G. This system is subjected to external surface loads f on Σ_f and to the action of gravity $\boldsymbol{g}$. By definition, the liquid free surface is initially orthogonal to gravity which is chosen to be in the opposite direction of the z-axis. Furthermore, the displacement of the structure is supposed to be known (equal to 0 for the sake of convenience) on Σ_u. The particular case of free fluid–structure systems has already been studied in previous works [11]. In this first section, we are going to write the coupled local equations of this system. The chosen variable for the structure is ordinarily its displacement field $\boldsymbol{u}^S$. The velocity field $\boldsymbol{v}^F$ is the natural variable to describe the fluid state. It is particularly suitable for flowing fluid, but in our case, the fluid is enclosed and to study its movement, we prefer to chose the displacement field $\boldsymbol{u}^F$ as variable.

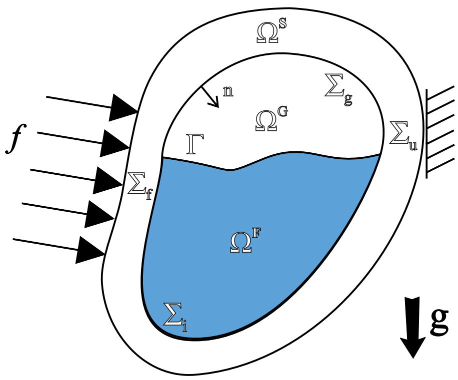

Fig. 1. Domain and external load definition.

2.1. Basic hypotheses and fluid model

As explained in the introduction, herein we are interested only in the linear vibrations of this system. We then suppose that external excitations are harmonic and seek stationary solutions of pulsation ω. Furthermore, we assume that the structure is elastic and the fluid is a Newtonian homogeneous liquid. Since we can consider the conservative associated system, we omit the damping aspect in this modelling and suppose that the liquid is inviscid (dynamic viscosity $\mu = 0$). In the frequency range of interest, the compressibility effects in the liquid can also be neglected which gives the relation

$$\boldsymbol{\nabla}.\boldsymbol{v}^F = \boldsymbol{0} \tag{1}$$

With all these assumptions, the Navier–Stokes equation written as

$$\rho^F\left(\frac{\partial \boldsymbol{v}^F}{\partial t} + \boldsymbol{\nabla}\frac{(\boldsymbol{v}^F)^2}{2} + (\boldsymbol{\nabla}\wedge\boldsymbol{v}^F)\wedge\boldsymbol{v}^F\right) = -\boldsymbol{\nabla}P + \rho^F\boldsymbol{g} + \mu\boldsymbol{\nabla}^2\boldsymbol{v}^F + \frac{\mu}{3}\boldsymbol{\nabla}(\boldsymbol{\nabla}.\boldsymbol{v}^F) \tag{2}$$

can be simplified and linearized with respect to the variable $\boldsymbol{v}^F$ to become the incompressible Euler equation

$$\rho^F\frac{\partial^2\boldsymbol{u}^F}{\partial t^2} = -\boldsymbol{\nabla}P + \rho^F\boldsymbol{g} \tag{3}$$

where ρ^F is the density of the liquid and P is the pressure in the liquid.

By taking the rotational of Eq. (3) and using the homogeneous property ($\boldsymbol{\nabla}\rho_F = \boldsymbol{0}$), we derive the following relation:

$$\rho^F\frac{\partial^2}{\partial t^2}(\boldsymbol{\nabla}\wedge\boldsymbol{u}^F) = \boldsymbol{0} \tag{4}$$

We then conclude that the irrotationality of the linear movements of such a liquid is a consequence of the previous assumptions (if we suppose that $\boldsymbol{\nabla}\wedge\boldsymbol{v}^F_{(t=0)} = \boldsymbol{\nabla}\wedge\boldsymbol{u}^F_{(t=0)} = 0$):

$$\boldsymbol{\nabla}\wedge\boldsymbol{u}^F = \boldsymbol{0} \tag{5}$$

2.2. Fluid local equations

Eq. (3) cannot be directly interpreted as the fluid vibratory equation, because, in this expression, $\boldsymbol{u}^F$ is