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On the supply function equilibrium and its applications in electricity markets

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Abstract

The paper deals with the Supply Function Equilibrium (SFE) as a model of competition in electricity markets. It introduces theoretical advancement through relaxing traditional assumptions of continuity of supply functions and provides a foundation for efficient computational algorithms. Two special examples are considered. One demonstrates that continuous equilibrium could be impossible while an infinite set of discontinuous equilibria exists. Another example proves the convergence to a linear equilibrium through learning in linear supply system. A possibility of a similar convergence for piece-wise linear system is being discussed.

Keywords: Supply function equilibrium; Competition in electricity markets; Strategic behavior; Market power

1. Introduction

The concept of supply function equilibrium (SFE) was originally developed by Klemperer and Meyer [11] as a way of modeling how competitors could achieve profit-maximizing equilibria in the marketplace under conditions of uncertain demand. The SFE approach was then adopted by Green and Newbery [9] as a model for strategic bidding in a competitive spot market for electricity. That particular publication attracted a substantial interest to the SFE model both in the industry and in academia. The SFE concept offers a compelling model of competitive behavior of multiple suppliers of a single product in which the existence of the Nash equilibrium does not require the demand to be elastic. Instead, representation of the suppliers' behavior by means of supply functions, rather than price-quantity pairs, creates elasticity of the residual demand faced by each player and could result in a sensible equilibrium outcome even if the demand is non-responsive to price. The on-going over the last decade deregulation of the electric industry in various countries of the world prompted industry analysts and consultants to study the SFE concept as means for modeling strategic behavior in electricity markets, creating tools for the direct analysis of market power, assessment of the impact of strategic behavior on electricity prices and on the

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market value of generating assets (see for example Ref. [14]).

Green and Newbery advanced the SFE theory by including capacity constraints [9] and by incorporating contracts for differences in the SFE framework, [7,8,12]. Rudkevich et al. [15] obtained a closed-form solution to the Klemperer–Meyer equation in a special case of zero price elasticity of demand, generalized the model for the case of non-convex step-wise marginal cost curves representing discrete generating units operating in the market. Anderson and Xu [2], following Anderson and Philpott [1], considered a similar problem but relied on a an original technique for representing supply functions as parametrical two-dimensional curves. This approach allowed them to obtain optimality conditions in a very general form allowing for a discontinuous relationship between price and quantity and to prove the existence of the optimal response of an individual player. Rudkevich [13] analyzed the ability of players to adapt their behavior through market observations and learning by means of the Cournot adjustment process and proved that players characterized with linear marginal costs and unconstrained capacities are capable of converging to the linear SFE. Baldick et al. [4] explored the applicability of this approach to piece-wise linear systems. Baldick and Hogan [3] attempted a substantial analytical and numerical explorations of SFEs in piece-wise linear systems.

Yet a wide spectrum of theoretical and applied problems remains unresolved. The original paper [11] was dealing with systems with uncertain demand and a single market clearing during the game period. Moreover, suppliers were assumed to be identical (symmetrical system). In light of these two assumptions, it is not surprising that the resulting equilibria are insensitive to the shape of the demand distribution (e.g., demand duration). Attempts to integrate Klemperer–Meyer equations in non-symmetrical cases could yield supply functions that were not always monotonically increasing or even always monotonically declining supply functions. In was not clear whether equilibrium conditions could be discontinuous and if so, what rules should be guiding their discontinuity. Finally, equilibrium conditions were explored only for the case of the so-called one-price payment rule.

In this paper, we offer a general formulation of the SFE game, consider a spectrum of payment rules ranging from the one-price to the pay-bid market design and derive necessary conditions of equilibria in those general settings. In all other parts we focus entirely on the system with one-price payment rule. We use these optimality conditions to explore equilibria in systems with zero marginal costs, analyze the adaptive learning process and its convergence to the equilibrium in linear systems, and outline an efficient algorithms for solving the problem of optimal response for piece-wise linear systems which could be used as a first step in adaptive learning in such systems. Finally, we briefly discuss the application of this algorithm to finding the equilibrium in supply functions for power systems.

2. Description of the SFE game

We consider a one-shot non-cooperative game of $n+1$ players—n competing generating firms and one market administrator (MA) such as an Independent System Operator (ISO) typically administering markets for electricity.

We assume that consumers' demand for the product is given in a form of a demand duration function $D(t,p)$. $D(t,p)$ depends on time and price such that $D_t'(t,p)>0$, $D_p'(t,p)\leq 0$. Time is continuous and $0\leq t\leq \bar{T}$. We further assume that firms are characterized by cost functions $C_j(q_j)$ that are continuous and piece-wise differentiable functions of production capacities q_j. Production capacities are bounded from below by zeros and from above by total capacity available to each firm: $0\leq q_j\leq W_j$; $j=\overline{1,n}$.

Prior to the beginning of the game period, each firm submits to the MA a supply function $q_j(p)$, which is a piece-wise differentiable, monotonically non-descending function of price. The MA develops a market-wide supply function

$$Q(p)=\sum_{j=1}^{n} q_j(p)$$

and in each moment of time $0\leq t\leq \bar{T}$ solves equation $Q(p)=D(t,p)$ for price. Solutions to this equation at each instantaneous moment $P(t)$ form a monotoni-