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# Four types of ignorance

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ABSTRACT

This paper studies alternative ways of representing uncertainty about a law of motion in a version of a classic macroeconomic targetting problem of Milton Friedman (1953). We study both "unstructured uncertainty" – ignorance of the conditional distribution of the target next period as a function of states and controls – and more "structured uncertainty" – ignorance of the probability distribution of a response coefficient in an otherwise fully trusted specification of the conditional distribution of next period's target. We study whether and how different uncertainties affect Friedman's advice to be cautious in using a quantitative model to fine tune macroeconomic outcomes.



## 1. Introduction

> "As Josh Billings wrote many years ago, "The trouble with most folks isn't so much their ignorance, as knowing so many things that ain't so." Pertinent as this remark is to economics in general, it is especially so in monetary economics." Milton Friedman (1965)[1]

Josh Billings may never have said that. Some credit Mark Twain. Despite, or maybe *because* of the ambiguity about who said them, those words convey the sense of calculations that Milton (Friedman, 1953) used to advise against using quantitative macroeconomic models to "fine tune" an economy. Ignorance about details of an economic structure prompted Friedman to recommend caution.

We use a dynamic version of Friedman's model as a laboratory within which we study the consequences of four ways that a policy maker might confess ignorance. One of these corresponds to Friedman's, while the other three go beyond Friedman's. Our model states that a macroeconomic authority takes an observable state variable $X_t$ as given and chooses a control variable $U_t$ that produces a random outcome for $X_{t+1}$:

$$X_{t+1} = \kappa X_t + \beta U_t + \alpha W_{t+1}. \tag{1}$$

The shock process $W$ is an iid sequence of standard normally distributed random variables. We interpret the state variable $X_{t+1}$ as a deviation from a target, so ideally the policy maker wants to set $X_{t+1} = 0$, but the shock $W_{t+1}$ prevents this.

Friedman framed the choice between "doing more" and "doing less" in terms of the *slope* of the response of a policy maker's decision $U_t$ to its information $X_t$ about the state of the economy. Friedman's purpose was to convince policy makers

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[1] From the forward to Phillip Cagan's *Determinants and Effects of Changes in the Stock of Money, 1875–1960*, Columbia University Press, New York and London, 1965. p. xxiii.

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to lower the slope. He did this by comparing optimal policies for situations in which the policy maker knows $\beta$ and in which it does not know $\beta$.

For working purposes, it is useful tentatively to classify types of ignorance into not knowing (i) response coefficients ($\beta$), and (ii) conditional probability distributions of random shocks ($W_{t+1}$). Both categories of unknowns potentially reside in our model, and we will study the consequences of both types of ignorance. As we will see, confining ignorance to not knowing coefficients puts substantial structure on the source of ignorance by trusting significant parts of a specification. Not knowing the shock distribution translates into not knowing the conditional distribution of $X_{t+1}$ given time $t$ information and so admits a potentially large and less structured class of misspecifications.

After describing a baseline case in which a policy maker completely trusts specification (1), we study the consequences of four ways of expressing how a policy maker might distrust that model[2]:

I. A "Bayesian decision maker" does not know the coefficient $\beta$ but trusts a prior probability distribution over $\beta$. (This was Friedman's way of proclaiming model uncertainty.)
II. A "robust Bayesian decision maker" uses operators of Hansen and Sargent (2007) to express distrust of a prior distribution for the response coefficient $\beta$. The operators tell the decision maker how to make cautious decisions by twisting the prior distribution in a direction that increases probabilities of $\beta$'s yielding lower values.
III. A "robust decision maker" uses either the multiplier or the constraint preferences of Hansen and Sargent (2001) to express his doubts about the probability distribution of $W_{t+1}$ conditional on $X_t$ and a decision $U_t$ implied by model (1). Here an operator of Hansen and Sargent (2007) twists the conditional distribution of $X_{t+1}$ to increase probabilities of $X_{t+1}$ values that yield low continuation utilities.
IV. A robust decision maker asserts ignorance about the same conditional distribution mentioned in item (III) by adjusting an entropy penalty in a way that Petersen et al. (2000) used to express a decision maker's desire for a decision rule that is robust at least to particular alternative probability models.

Approaches (I) and (II) are ways of 'not knowing coefficients' while approaches (III) and (IV) are ways of 'not knowing a shock distribution.' We compare how these types of ignorance affect Friedman's conclusion that ignorance should induce caution in policy making.[3]

## 2. Baseline model without uncertainty

Following Friedman, we begin with a decision maker who trusts model (1). The decision maker's objective function at date zero is

$$-\frac{1}{2}\sum_{t=0}^{\infty}\exp(-\delta t)E\left[(X_t)^2|X_0=x\right] = -\frac{1}{2}\sum_{t=0}^{\infty}\exp\left[-\delta(t+1)\right]E\left[(\kappa X_t+\beta U_t)^2|X_0=x\right]-\frac{1}{2}x^2-\frac{\alpha^2\exp(-\delta)}{2[1-\exp(-\delta)]}, \tag{2}$$

where $\delta>0$ is a discount rate. The decision maker chooses $U_t$ as a function of $X_t$ to maximize (2) subject to the sequence of constraints (1). The optimal decision rule

$$U_t = -\frac{\kappa}{\beta}X_t \tag{3}$$

attains the following value of the objective function (2):

$$-\frac{1}{2}x^2-\frac{\alpha^2\exp(-\delta)}{2[1-\exp(-\delta)]}.$$

Under decision rule (3) and model (1), $X_{t+1}=\alpha W_{t+1}$.

In subsequent sections, we study how two types of ignorance change the decision rule for $U_t$ relative to (3):

- Ignorance about $\beta$.
- Ignorance about the probability distribution of $W_{t+1}$ conditional on information available at time $t$.

## 3. Friedman's Bayesian expression of caution

This section sets Friedman's analysis within a perturbation of model (1). We study how the decision maker adjusts $U_t$ to offset adverse effects of $X_t$ on $X_{t+1}$ when he does not know the response coefficient $\beta$. Does he do a lot or a little? Friedman's purpose was to advocate doing less relative to the benchmark rule (3) for setting $U_t$.

[2] Our preoccupation within enumerating types of ignorance and ambiguity here and in Hansen and Sargent (2012) is inspired by Epson (1947).
[3] Approaches (I), (II), and (III) have been applied in macroeconomics and finance, but with the exception of Hansen and Sargent (2015), approach (IV) has not.