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Sequential voting in large elections with multiple candidates

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ABSTRACT

I analyze strategic voting incentives in large elections with three candidates when voting takes place sequentially. Voters have perfect information about their private preferences but do not know the distribution from which other voters' preferences are drawn. If a candidate finishes last in an early voting round, voters deduce that this candidate is likely to be less popular amongst the remaining voters, and the remaining voters almost always have an incentive to stop voting for this candidate. By contrast, sincere voting equilibria can exist under either simultaneous voting or an early voting round of sequential voting without knife-edge assumptions.

1. Introduction

A common feature of many electoral systems is that some voters cast their ballots after learning how other voters have already voted before them. One of the most prominent examples of this is American presidential primaries, in which voters in certain states cast their votes after observing how voters in other states have voted. Presidential primaries often begin with several serious candidates, but as the election progresses voters tend to mostly vote for one or two candidates who have done particularly well in early voting rounds. This winnowing of candidates results when candidates that have done poorly in early voting rounds are eliminated from consideration (Aldrich, 1980; Bartels, 1988; Matthews, 1978).[1]

Why would voters in later states stop voting for candidates who have done poorly in early voting rounds? One view might be that voters have uncertainty about the underlying quality of the candidates but voters infer that if a candidate did poorly in an early voting round, then the candidate is less likely to be a high quality candidate and voters stop voting for this candidate. Deltas and Polborn (2009), Deltas et al. (2011), and Knight and Schiff (2010) empirically analyze models in which voters make inferences about the quality of the candidates as a result of early voting rounds. Another theory might be that even if voters have perfect information about their preferences over candidates, then voters would still stop voting for candidates who performed poorly in early voting rounds because they vote strategically by only voting for candidates who have the best chance of winning. Abramson et al. (1992) and Popkin (1994) both suggest that the fact that at most two candidates garner significant vote shares in later voting rounds may be evidence of strategic voting by voters who wish to avoid wasting a vote on a losing candidate.

This paper presents a formal model of strategic voting that is consistent with the fact that only candidates who do well initially can expect to obtain more votes from later voters. I consider a two-period model of sequential voting with three candidates in which voters know their own private preferences but not those of the other voters. Departing from standard models of strategic voting, I assume that voters do not have perfect information about the precise probabilities with which the other voters like the given candidates best.

Since the probabilities with which voters like certain candidates best are not known with certainty, voters can learn about these probabilities by observing the results of the early voting round. Thus voters in the later voting round can better anticipate the likely private preferences of future voters by using the results from the early voting round. This in turn enables voters in the later voting round to better deduce which of the three candidates have serious chances of winning.

Because voters in the later voting round will have a better idea of how their vote is most likely to be pivotal after the early voting round, these voters will normally condition their votes on the results of the early voting round. One of the main results of the paper is that voters

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[1] One can find additional empirical examples of presidential primaries in which voters have voted differently because of the results of early voting rounds in Bartels (1985) and Kenney and Rice (1994).

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almost always have an incentive to stop voting for a candidate who finishes last in the early voting round, thus giving a dynamic version of Duverger's (1954) Law. This is consistent with the empirical observation that candidates who perform poorly in early rounds of sequential voting are winnowed out of the election.

I also consider how the assumption that there is uncertainty about the distribution from which voter preferences are drawn affects the existence of sincere voting equilibria under either simultaneous voting or the early round of sequential voting. In either of these circumstances, there exist sincere voting equilibria without knife-edge assumptions regarding the form of the uncertainty about the distribution from which voter preferences are drawn. I further address when voters would have an incentive to vote sincerely when the form of uncertainty about the distribution of voter preferences corresponds to that which would be induced by a pre-election poll on voter preferences.

I focus on sincere voting equilibria because a large empirical literature in political economics addresses questions related to when voters vote sincerely or strategically in large multicandidate elections (Alvarez and Nagler, 2000; Bensel and Sanders, 1979; Burden, 2005; Johnston and Pattie, 1991; Lanoue and Bowler, 1992; Shively, 1970). For this reason, it seems important to also understand theoretically when we would expect sincere voting to occur in elections with several candidates. The results I derive about sincere voting equilibria enable me to compare theoretical circumstances under which voters could vote sincerely in equilibrium with empirical conclusions on when sincere voting takes place. The circumstances under which voters have an incentive to vote sincerely seem consistent with empirical evidence regarding when voters vote sincerely in early voting rounds of US presidential primaries.

There has been little work on why rational voters would condition their votes on the results of early voting rounds and choose to vote for candidates that have done well early on. Dekel and Piccione (2000) have shown that, in elections with two candidates, informative equilibria under simultaneous voting are also equilibria under sequential voting. Sequential voting therefore need not cause voters to adopt different strategies than they would employ under simultaneous voting. Ali and Kartik (in press), Callander (2007), and Hummel (2011a) consider circumstances under which voters might employ history-dependent strategies when there are two candidates, but do not consider why an election with more than two candidates might winnow down to the two candidates who do best in early voting rounds.[2] Morton and Williams (1999, 2001) consider elections with three candidates, but focus on whether simultaneous or sequential voting is more likely to elect a Condorcet winner. Other papers on sequential voting (e.g., Aldrich (1980), Battaglini (2005), Battaglini et al. (2007), Klumpp and Polborn (2006), Strumpf (2002)) focus on different issues than why voters would choose to vote for candidates that have done well in early voting rounds.[3]

My paper explains the fact that elections with sequential voting typically winnow from several candidates to the two candidates that do best early on with the assumption that the distribution from which private preferences are drawn is not known to the voters. Such an assumption is realistic, since the distribution from which voter preferences are drawn is rarely known with certainty in large elections. Nonetheless, this assumption is surprisingly rare in the literature. I only know of a few other theory papers which make use of this assumption in games with a finite number of agents (Chamberlain and Rothschild, 1981; Ekmekci, 2009; Evren, 2010; Good and Mayer, 1975; Mandler, 2010, in press; Myatt, 2007), and these papers all address questions unrelated to sequential voting.

2. The model

There are three candidates A, B, and C, and a set of voters $N=\{1,\ldots,2n\}$. Each voter has strict preferences over all three candidates. A voter obtains a utility of 1 if his or her preferred candidate is elected, a utility of 0 if his or her second favorite candidate is elected, and a utility of -1 if his or her least favorite candidate is elected. Each voter knows his or her private preferences but does not know the private preferences of any of the other voters.

Each voter's preferred candidate is an independent and identically distributed draw from a distribution that takes on the value A with probability p_A, B with probability p_B, and C with probability $p_C=1-p_A-p_B$. However, the precise values of p_A, p_B, and p_C are not known to the voters. Before each voter's favorite candidate is drawn from the distribution, the values of p_A and p_B are themselves drawn from another distribution with a continuously differentiable density $f(p_A,p_B)$. $f(p_A,p_B)$ is assumed to be strictly positive on the interior of $\Delta=\{(p_A,p_B)|p_A\geq 0,p_B\geq 0,p_A+p_B\leq 1\}$ and zero outside Δ.

Given this uncertainty about the distribution from which private preferences are drawn, a player's expectations about this distribution depend on his or her private preferences. For instance, a player whose favorite candidate is A would think it is relatively more likely that the state of the world is one in which p_A is relatively larger. If a player's preferred candidate is A, then the player believes the density at (p_A,p_B) is $f_A(p_A,p_B)=p_Af(p_A,p_B)/\int_0^1\int_0^{1-p_B}p_Af(p_A,p_B)dp_Adp_B$.

I consider strategic voting behavior under both simultaneous and sequential voting. Under simultaneous voting, all $2n$ voters cast their votes at the same time without knowing how other voters are voting. Under sequential voting, first voters $1,\ldots,n$ vote for a candidate without knowing how others have voted, and then voters $n+1,\ldots,2n$ vote for a candidate after observing how voters $1,\ldots,n$ have voted. Sequential voting thus represents a succession of two large electoral contests such as Super Tuesday.

In either simultaneous or sequential voting, if a candidate receives a strictly larger number of votes than the other two candidates, then that candidate is elected. If two candidates receive the same number of votes and these candidates both receive a strictly larger number of votes than the other candidate, then each of these two candidates is elected with probability $\frac{1}{2}$. Finally, if all three candidates receive the same number of votes, then each candidate is elected with probability $\frac{1}{3}$.

3. Simultaneous voting

Suppose for the time being that a player's favorite candidate is A, a player's second favorite candidate is B, and a player's least favorite candidate is C. Note that voting for A weakly dominates voting for C, so the only interesting question is whether the player should vote for A or B. I thus wish to compare the relative benefits of voting for A with the relative benefits of voting for B.

There are three basic types of circumstances in which a player's vote can affect the player's payoff. The first circumstance is when A and B receive nearly identical vote shares but receive higher vote shares than C. In this case a player's vote may affect whether candidate A or B is elected. Another situation is when A and C receive nearly identical vote shares but receive higher vote shares than B. In this case a player's vote may affect whether candidate A or C is elected. The last type of situation in which a player's vote may affect his payoff is when B and C receive nearly identical vote shares but receive higher vote shares than A. In this case a player's vote may affect whether candidate B or C is elected.

If the player knew that he were in one of the first two types of circumstances, then the player would strictly prefer voting A to B, as a

[2] Hummel (2011a) shows how these history-dependent strategies arise in binary elections when voters prefer that the election end as quickly as possible.

[3] In addition, Castanheira (2003) and Piketty (2000) present models which analyze strategic voting when voters vote in two separate elections that take place sequentially.