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Dam-break release of a gravity current in a stratified ambient

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Abstract

The 'dam-break' initial behaviour of an inviscid gravity current which is released from a lock and then propagates over a horizontal boundary at the base of a stratified ambient fluid is considered. Analytical and finite-difference solutions of the one-layer shallow-water equations are developed and compared for the linear stratification in a rectangular channel case, and corroborated by numerical solutions of the full two-dimensional Navier–Stokes equations. Extensions of the shallow-water solution to non-linear stratification, release from an elliptical reservoir, and axisymmetric geometry are also presented. The results indicate that the shallow-water formulation captures well the essential features of the motion, which are qualitatively similar to the non-stratified case, but with details modified by the stratification; in particular, the forward propagation of the head and the backward spread of the depression wave are reduced when the stratification increases.

Keywords: Gravity current; Stratified; Dam-break; Shallow-water; Finite-differences

1. Introduction

The dam-break flow of a water reservoir open to the atmosphere is a fundamental, well known and extensively researched problem in hydraulics, starting with Saint-Venant's solution in 1843, see for example Billingham and King [1]. A closely related fundamental problem appears in the study of the gravity current phenomena which are of interest in a wide range of industrial and geophysical application (Simpson [2], Huppert [3]). Indeed, the generation of a gravity current by the release of a stationary volume of fluid of given density from behind a lock into an ambient fluid of a (slightly) different density is a dam-break type problem in which the embedding fluid has some influence on the major motion of the released fluid. First, the 'reduced gravity' (based on the relative density difference) replaces the full gravitational acceleration of the classical water-air configuration; and second, the gravity current is subject to an additional condition, namely, that the front of the propagating fluid (i.e., the nose of the current) advances like a "wall" in the embedding fluid, see Klemp et al. [4]. An accepted versatile formulation of the gravity current flow is via the inviscid shallow-water (SW) approximation, and the dam-break problem provides elegant and insightful analytical solutions to these equations. This problem is therefore essential to the understanding of the initial motion of a gravity current and an efficient tool for testing numerical solvers of the SW equations. However, no analytical investigations of the dam-break problem for the gravity current in a stratified ambient have been presented, to our best knowledge. We think it is both of academic and practical importance to close this gap of knowledge concerning this fundamental problem.

Recent investigations of gravity currents in a (linearly) stratified ambient were presented by Maxworthy et al. [5] and by Ungarish and Huppert [6], hereafter referred to as UH. The former presented experimental measurements and numerical

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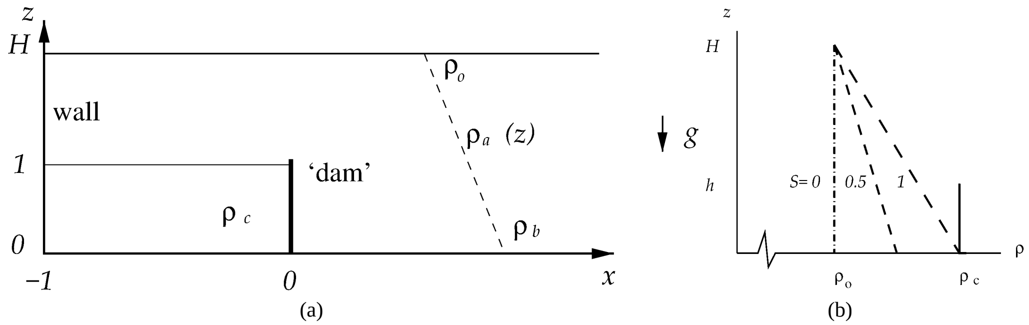

Fig. 1. Schematic description of the system at $t = 0$. (a) The geometry. Here the horizontal lengths are scaled with the length of the lock, x_0, and vertical lengths are scaled with the height of the lock, h_0. (In the elliptic lock system the dense fluid is initially in the domain $0 \leqslant z \leqslant (-x^2 - 2x)^{1/2}$, $-1 \leqslant x \leqslant 0$.) (b) Density profiles in the current (solid line) and ambient, for linear stratification at various values of S (dashed lines).

simulations. UH developed the one-layer SW equations and the nose boundary condition, and obtained solutions for the nose velocity, u_N, during the slumping phase of propagation with constant velocity. These theoretical results concerning the nose velocity are in very good agreement with the experiments of Maxworthy et al. [5], see Fig. 6 and Section 3 of UH. UH did not present SW solutions of the current, and therefore a logical step to advance the previous theoretical-analytical work is the investigation of the dam-break problem. This study requires a detailed solution of the SW equations during a significant time interval following the release from behind the lock, for which the previously calculated u_N will serve as a reliable boundary condition. The analysis is expected to provide insights into the modeling and understanding of the time-dependent constant-volume gravity current (or intrusion) in a stratified ambient. We notice that, to our best knowledge, no reliable theoretical predictive tools are available for the analysis of this phenomenon, and we think it would be beneficial to close the gap with the non-stratified well developed counterpart.

These considerations motivated the present work, whose combined objectives are (a) to extend the dam-break problem to a gravity current in a stratified ambient, and (b) to extend the study of UH to the initial motion of the entire gravity current via the solution of the SW equations, and also to non-linear stratifications.

The system under consideration is sketched in Fig. 1: a layer of ambient fluid of height H and (stable) density $\rho_a(z)$, lies above the horizontal surface $z = 0$. Gravity acts in the $-z$ direction. In the rectangular case considered here the system is bounded by parallel vertical smooth impermeable surfaces and the current propagates in the direction labeled x. A given volume of homogeneous fluid of density $\rho_c \geqslant \rho_a(z = 0) \equiv \rho_b$ and kinematic viscosity ν, is initially at rest in a rectangular box of height h_0 and length x_0, bounded by a 'dam' (or gate) at $x = 0$ and a solid wall at $x = -x_0$. We assume that the height of the dense fluid does not exceed that of the ambient. At time $t = 0$ the dam is instantaneously removed, and a two-dimensional current commences to spread. We assume that the Reynolds number of the horizontal flow, *Re*, defined below, is large and hence viscous effects can be neglected. We attempt to predict the shape (position of nose and upper interface) of the dense fluid (current) and the main (horizontal) velocity field within.

The structure of the paper is as follows. In Section 2 the shallow-water equations of motion and the appropriate boundary conditions are developed. Analytical and finite-difference solutions for the dam-break stage are presented in Section 3, for linear and non-linear stratifications. Some numerical results of the Navier–Stokes equations are also presented for corroboration of the SW predictions. The release from the classical rectangular reservoir is briefly contrasted with release from an elliptical container. In Section 4 concluding remarks are given. The non-stratified counterpart results are summarized in Appendix A, and a brief extension to the axisymmetric configuration is provided in Appendix B.

2. Formulation and shallow-water (SW) approximation

The configuration is sketched in Figs. 1 and 2. We use a $\{x, y, z\}$ Cartesian coordinate system with corresponding $\{u, v, w\}$ velocity components, and assume that the flow does not depend on the coordinate y and that $v \equiv 0$. Initially, the height of the released current is h_0, its length is x_0 and the density is ρ_c. The height of the ambient fluid is H and the density in this domain decreases with z from ρ_b to ρ_o. (The subscripts b, o refer to bottom and open surface values.)

It is convenient to use ρ_o as the reference density and to introduce the reduced density differences

$$\epsilon = \frac{\rho_c - \rho_o}{\rho_o}, \qquad \epsilon_b = \frac{\rho_b - \rho_o}{\rho_o}, \tag{2.1}$$