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Drag force acting on a bubble in a cloud of compressible spherical bubbles at large Reynolds numbers

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Abstract

We have derived the expressions for the viscous forces acting on a bubble in a cloud of bubbles by using the approach by Levich. To obtain the dissipation function, an approximate expression for the velocity potential calculated previously by the authors up to order β^3 has been used. Here $\beta = \bar{b}/d$ is a small dimensionless parameter, $\bar{b}$ is the mean bubble radius and d is the mean distance between bubbles.

Keywords: Bubbly fluids; Lagrange's formalism; Levich's approach

1. Introduction

Levich [1] calculated the drag force acting on a bubble moving through a liquid at large Reynolds and small Weber numbers (the last condition guarantees that the bubble shape is spherical) defining the total viscous dissipation through the velocity potential of irrotational flow (see also Moore [2] and Batchelor [3]). With the use of this approach, he obtained the drag force F in a translational motion of a bubble of unchanged radius

$$F = 12\pi\mu bU. \tag{1}$$

It differs from the drag force given by the theory of viscous potential motions. In the last method, the viscous flow is supposed to be potential, and the viscosity is taken into account only in the dynamic condition expressing the continuity of the normal stress at the gas–liquid interface (Moore [4], see also Joseph and Wang [5]). The corresponding drag force is then

$$F = 8\pi\mu bU.$$

Here μ is the dynamic viscosity of the fluid, U is the bubble velocity, and b is the bubble radius. Moore [2] justified further the approach by Levich by developing of the model of the boundary layer wrapped around bubble. Moreover, recently Magnaudet and Legendre [6] calculated the drag force on a spherical compressible bubble by using the full Navier–Stokes equations. In

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particular, they found that the Levich formula (1) is valid not only in the high-Reynolds limit, but also for moderate and even small Reynolds numbers, provided the bubble pulsation velocity is high with respect to the flow velocity.

Both approaches, the method of dissipation function and the method of viscous potential motions, give the same result if we calculate the viscous forces on an oscillating bubble without translational motion (the Rayleigh–Plesset equation):

$$b\frac{\mathrm{d}^2 b}{\mathrm{d}t^2}+\frac{3}{2}\left(\frac{\mathrm{d}b}{\mathrm{d}t}\right)^2=\frac{1}{\rho}(p_g-p_\infty)-\frac{4\mu}{\rho b}\frac{\mathrm{d}b}{\mathrm{d}t}. \tag{2}$$

Here ρ is the fluid density, $p_g\,(b)$ is the pressure in a bubble (it is a given function of b), and p_∞ is the fluid pressure at infinity.

The Levich approach allows to apply the Lagrange formalism for constructing the governing equations of motion with non-potential forces derived from the Rayleigh dissipation function method (see, for example, Goldstein, Poole and Safko [7]). More exactly, if we consider a bubble of radius $b(t)$ oscillating with the velocity $s(t)=\mathrm{d}b/\mathrm{d}t$ whose center position is $\mathbf{r}(t)$ and the translational velocity $\mathbf{v}(t)=\mathrm{d}\mathbf{r}(t)/\mathrm{d}t$, the equations of motion are (Voinov and Golovin [8]):

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \mathbf{v}}\right)-\frac{\partial L}{\partial \mathbf{r}}=-\frac{1}{2}\frac{\partial \Phi}{\partial \mathbf{v}},\\&\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial s}\right)-\frac{\partial L}{\partial b}=-\frac{1}{2}\frac{\partial \Phi}{\partial s}.\end{aligned} \tag{3}$$

Eqs. (3) imply the energy equation in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \mathbf{v}}\mathbf{v}+\frac{\partial L}{\partial s}s-L\right)=-\Phi.$$

Here L is the Lagrangian of the system, Φ is the Rayleigh dissipation function. In the case of a massless bubble, the Lagrangian is

$$L=\frac{\pi\rho b^3}{3}|\mathbf{v}|^2+2\pi\rho b^3 s^2-\varepsilon(\tau).$$

Here

$$\varepsilon(\tau)=\int_\tau^\infty p_g(z)\,\mathrm{d}z+p_\infty\tau$$

is the internal energy of the gas–liquid system, $\tau=4\pi b^3/3$ is the bubble volume. The dissipation function Φ can be explicitly calculated for the velocity potential of the flow around a single bubble:

$$\varphi=-\frac{b^2 s}{|\mathbf{x}-\mathbf{r}(t)|}+\frac{b^3}{2}\mathbf{v}\cdot\nabla\left(\frac{1}{|\mathbf{x}-\mathbf{r}(t)|}\right).$$

It is given by

$$\Phi=2\mu\int_{R^3\setminus B}D:D\,\mathrm{d}\omega=-\mu\int_\Gamma\frac{\partial}{\partial n}|\nabla\varphi|^2\,\mathrm{d}\Gamma=4\pi\mu b\left(4s^2+3|\mathbf{v}|^2\right). \tag{4}$$

Here B is a region occupied by the bubble, $\Gamma=\partial B$ is the bubble boundary, $D=(\partial\mathbf{u}/\partial\mathbf{x}+(\partial\mathbf{u}/\partial\mathbf{x})^{\mathrm{T}})/2$ is the rate of deformation tensor, $\mathbf{u}=\nabla\varphi$, $\mathbf{n}$ is the normal vector to Γ directed to the fluid. System (3) can be rewritten in the following equivalent form:

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{2}{3}\pi\rho b^3\mathbf{v}\right)=-12\pi\mu b\mathbf{v},\\&b\frac{\mathrm{d}^2 b}{\mathrm{d}t^2}+\frac{3}{2}\left(\frac{\mathrm{d}b}{\mathrm{d}t}\right)^2-\frac{1}{4}|\mathbf{v}|^2=\frac{1}{\rho}\left(p_g(b)-p_\infty\right)-\frac{4\mu}{\rho b}\frac{\mathrm{d}b}{\mathrm{d}t}.\end{aligned} \tag{5}$$

The general case of N interacting rigid bubbles was considered first by Golovin [9]. For a dilute bubbly mixture, he derived the equations of motion as well as the friction forces (by using Levich's approach) up to order β^3. Here $\beta=\bar{b}/d$ is a small parameter which is the ratio of the mean bubble radius $\bar{b}$ to the mean distance d between bubbles. Obviously, $4\pi\beta^3/3$ is the volume fraction of bubbles in the limit $N\to\infty$. Kok [10,11] (and references therein), studied analytically and experimentally the dynamics of a pair of rigid bubbles of equal radii. In particular, he derived the equations of motion up to any order of β. Sangani and Didvania [12] and Smereka [13] examined the dynamics of N bubbles numerically. In particular, they showed that