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Transition to instability of the interface in geothermal systems ☆

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Abstract

High-temperature geothermal reservoir in porous media is under consideration, consisting of two high-permeability layers, which are separated by a low-permeability stratum. The thermodynamic conditions are assumed to imply that the upper and lower high-permeability layers are filled in by water and by vapour, respectively. In these circumstances the low-permeability stratum possesses the phase transition interface, separating domains occupied by water and vapour. The stable stationary regimes of vertical phase flow between water and vapour layers in the low-permeability stratum may exist. Stability of such regimes where the heavier fluid is located over the lighter one is supported by a heat transfer, caused by a temperature gradient in the Earth's interior. We give the classification of the possible types of transition to instability of the vertical flows in such a system under the condition of smallness of the advective heat transfer in comparison with the conductive one. It is found that in the non-degenerate case there exist three different scenarios of the onset of instability of the stationary vertical phase transition flows. Two of them are accompanied by the bifurcations of the destabilizing vertical flow, leading to appearance of horizontally non-homogeneous regimes with non-constant shape of the interface. The bifurcations correspond to the simple resonance and 1 : 1-resonance, which typically arise in reversible systems.

Keywords: Geothermal system; Phase transition interface; Threshold of instability; Bifurcation

1. Introduction

In the framework of classical fluid dynamics the equilibrium of heavy fluid over the lighter one is always unstable. The instability in question is known as the Rayleigh–Taylor instability (see e.g. Chandrasekhar, [9]). Yet, when supplementary physical mechanisms have to be taken into consideration in a hydrodynamic model, such an equilibrium is possible. Geothermal reservoir gives an example of the natural system where the thermodynamic states are realized, supporting the stable existence of water (the heavier fluid) over vapour (the lighter fluid). Full-scale investigations of natural geothermal systems showed, that in a great number of reservoirs the situation takes place, when a water layer of a considerable thickness is located over a layer of superheated vapour [1,2]. The existence of such a configuration is explained from the thermodynamic point of view by a

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considerable temperature gradient typically taking place in geothermal system. In this case the thermodynamic conditions at large depths correspond to vapour, and at smaller depths – to water.

Schubert and Straus [3] treated the example of the geothermal reservoir of the homogeneous vertical extend embedded in the rigid rock. In such a reservoir the basic state describes the motionless equilibrium of the water layer over the vapour one. The stability of such a configuration for some range of values of the permeability of the geothermal reservoir was shown by means of numerical analysis of the dispersion relation.

Tsypkin and Il'ichev [4] analysed the more general case of a geothermal system in a sense that the phase motion and phase transition in a base state are permitted. A high-temperature geothermal reservoir was considered, consisting of two high-permeability layers, which are separated by a low-permeability stratum. It was assumed that the thermodynamic conditions imply that the upper and lower high-permeability layers are filled in by water and by vapour, respectively. Then in the low-permeability layer there exists the phase transition interface, separating domains occupied by water and vapour. The geometry of the reservoir is presented in Fig. 1. The reservoir itself represents the fragment of the general reservoir, based on geologocal structure consisting of blocks of lower permeability embedded in the base rock [2]. In dependence on values of pressure in the high-permeability layers, either the regime of vapourization, when the water moves downwards, or the regime of condensation, corresponding to the vapour motion upwards, take place. In the rest state phase transitions are absent, and the pressure distribution in the high-permeability layers coincides with the hydrostatic one.

Our principal restriction is that we assume that the conductive heat transfer in the system considerably exceeds the advective one. This restriction, nevertheless, permits to treat a lot of natural geothermal systems. This statement is based on the following estimates. For the water domain, combining Darcy's law with the heat conservation equation, we obtain the dimensionless parameter that specifies the ratio of advective and conductive terms [4]:

$$\frac{\rho_w C_w}{\mu_w \lambda_1} k(\delta P - \rho_w g l), \tag{1.1}$$

where l is the characteristic length scale and δP is the dynamical pressure variation. For other notations see table at the beginning of Section 2 of the paper. Permeability and pressure may vary strongly, at the same time the other physical parameters in (1.1) vary slightly. Therefore, after substitution of the characteristic values of parameters in (1.1) the condition of smallness of advective transfer in the water domain can be written in the following form

$$k|\delta P - \rho_w g l| \ll 10^{-10}\ \mathrm{N}. \tag{1.2}$$

Analogously, one has for the vapour domain

$$k|\delta P - \rho_v g l| \ll 3 \cdot 10^{-9}\ \mathrm{N}. \tag{1.3}$$

The criteria (1.2), (1.3) occur to be valid for a lot of flows in natural geothermal systems.

Calculations showed that there exist the regimes of motion with phase transitions, when the water layer over the vapour layer in geothermal system occurs to be stable also for permeability values which exceed the value got in Schubert and Straus [3] by the order of magnitude. The new threshold value of the permeability explains the fact of stability of a lot of geothermal systems, observed in nature.

The character of the arising secondary flows depends on the type of instability and, therefore, it seems to be important to investigate in detail the possible types of transition to instability. In this paper we present the results of the analytic investigation of stability of a flow in the geothermal system treated by Tsypkin and Il'ichev [4]. The criteria of the transition to instability satisfying the principle of exchange of stabilities (when the time exponent of the least stable normal mode is zero at the margin of stability) are obtained in explicit form. It is established that the transition to instability of the interface under the variation of physical parameters takes place by means of one of the following four mechanisms:

- comes spontaneously for all wave numbers of perturbations (degenerate case);
- unstable wave number arises at infinity;
- a threshold of instability is determined by double-zero wave number;
- a threshold of instability is determined by a pair of multiple non-zero wave numbers.

In the two last cases the transition to instability is accompanied by bifurcations of the simple resonance and 1 : 1-resonance, respectively. These bifurcations lead to the branching of base regimes, describing horizontally homogeneous vertical phase flows, and appearance of secondary regimes depending on the horizontal coordinate.

The paper organized as follows. In Section 2 we give formulation, describe base regimes and present the dispersion relation. In Section 3 the types of transition to instability in the degenerate case (when the phase transition interface of the base regime is located exactly in the middle of the low-permeability stratum) and in the general case of location of the interface are described. In Section 4 we discuss the results and give our conclusions.