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Proportional control of oscillatory thermocapillary convection in a toy model

J. Shiomi, G. Amberg *

KTH Mechanics, Stockholm, S-100 44, Sweden

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Abstract

Simple model equations were formulated to examine the influence of linear feedback control on oscillations in thermocapillary convection. Limiting the solutions to have a few wavenumbers and roughly assuming the other spatial profiles, the system of equations is reduced to a set of ordinary differential equations. This toy model is able to recreate basic features of the uncontrolled system such as standing/traveling wave structures and bifurcation characteristics. The control was realized by locally heating and cooling the free surface by linearly feeding back the temperature signals measured in local positions. Implementing the proportional feedback control in this toy model, we could capture some of the essential qualitative features of the influence of the control observed in the previous experiments, including the limitation of the control. The formulation of the toy model can be used to gain physical insight in the control problem.

Keywords: Thermocapillary convection; Feedback; Control; Model; Bifurcation

1. Introduction

Oscillatory thermocapillary convection is often blamed for detrimental striations in the finished material produced by the container-less crystal growth method called floating-zone method [1]. Having the industrial motivation and advantage to be carried out in a micro-gravity condition, there have been a growing number of researches reported on the oscillatory thermocapillary convection in the past decades. Since the first experimental observation of the three-dimensional time-dependent state in thermocapillary convection by Schwabe and Scharmann [2] and Chun and Wuest [3], many works have been devoted to identify the mechanism of the instability [4–6] and to reveal the modal structures at the onset of the oscillation and their bifurcation characteristics in the supercritical regime [7,8]. Recently, flow structures for considerably high Marangoni number (*Ma*) are reported, where the flow becomes chaotic and turbulent-like [9].

With the knowledge obtained from those studies concerning the instability mechanism and bifurcation characteristics, the ultimate goal would be to suppress the oscillation. Our challenge in this report is to stabilize the flow by local modification of heat conduction on the free surface based on continuous feedback control. Some works have been reported on the control of oscillatory thermocapillary convection in various geometries.

* Corresponding author.
E-mail addresses: shiomi@mech.kth.se (J. Shiomi), gustava@mech.kth.se (G. Amberg).

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An attempt to stabilize the thermocapillary wave instability in an experiment on a plane fluid layer was made by Benz et al. [10]. The temperature signal and the phase information sensed by thermocouples near the cold end of the layer were fed forward to control a laser which heated the downstream fluid surface along a line.

A nonlinear control was performed by Petrov et al. [11,12] to stabilize the oscillation in a half-zone model by using local temperature measurements close to the free surface and modifying the temperature at different local locations. They have constructed a look-up table based on the system's response to a sequence of random perturbations. A linear control law using appropriate data sets from the look-up table was computed. The control law was updated at every time step to adapt the control law to the nonlinear system. Using one sensor/actuator pair, a successful control was observed at the sensor location for $Ma \sim 17\,750$, however infrared visualization revealed the presence of standing waves with nodes at the feedback element and the sensor. This was resolved by adding a second sensor/actuator pair, which allows the control to damp out both waves propagating clockwise and counterclockwise, thus standing waves. The performance of the control was reported for a fixed $Ma \sim 15\,000$, where the critical value was $Ma_{\mathrm{cr}} \sim 14\,000$.

This was followed by studies applying linear and weakly nonlinear control. For an annular configuration, Shiomi et al. [13,14] applied active feedback control based on a simple cancellation scheme. Active control was realized by locally modifying the surface temperature using the local temperature measured at different locations fed back through a simple control law. Using two sensor/actuator pairs, a significant attenuation of the oscillation was observed in a range of Ma, with the best performance in the weakly nonlinear regime. Applying the control on an oscillation with azimuthal wave number of 3 (mode-3), in the regime with weak nonlinearity, the oscillation was suppressed to the background noise level. The experiments also revealed the limitation of the control. When Ma is about 15% above the critical value, control fails to achieve complete suppression of the oscillation, though a significant attenuation is still achieved. The loss of control is accompanied by an increase in the amplitude of the first overtones and a modulation in the controlled signal, which may suggest the appearance of another mode triggered by the control.

Recently, with a similar method, but in a half-zone model, weakly nonlinear control of the oscillatory thermocapillary convection is reported by Shiomi et al. [15]. The experiment utilizes a unit aspect ratio liquid bridge where the most dangerous mode has an azimuthal wave number of 2 when the control is absent. The performance of control was quantified by analyzing local temperature signals and the flow structure was simultaneously identified by flow visualization. With optimal placement of sensors and heaters, proportional control can raise Ma_{cr} by more than 40%. The amplitude of the oscillation can be suppressed to less than 30% of the initial value up to 90% of Ma_{cr}. The proportional control was tested for a period doubling state to stabilize the oscillation to a periodic state. Weakly nonlinear control was applied by adding a cubic term in the control law to improve the performance of the control and to alter the bifurcation characteristics of the system.

Our earlier experimental works in proportional control have shown not only successful performance but also its limitation. In the studies in an annular geometry by Shiomi et al. [13,14], it was observed that the proportional control performs worse as the nonlinearity of the system becomes stronger. The results suggest that the limitation of control may be due to the appearance of the neighboring modes, however, this solely does not explain why the overtones are amplified. There seems to be a mechanism in the controlled system to amplify the overtones. In order to investigate the cause of the limitation and the possibility of a remedy, a toy model was formulated. Here, the intention is to construct a toy model which, at least qualitatively, reproduces important linear and nonlinear features of the system. For this purpose, limiting the number of azimuthal modes to the fundamental and first harmonic ones, and assuming the other spatial profiles, we formulate a set of ordinary differential equations. Consequently, implementing the feedback control to this toy model, we could capture some of the essential qualitative features of the influence of the control.

The model equations are addressed as *toy models* due to the extreme simplification based on rather crude assumptions. Being a *toy*, there are possibilities of discrepancies with the actual system which can certainly result in the limitation of the toy model for further application. However, such an approach is useful to reduce the complexity of the modeled system and to grow insight to the problem. Especially, when certain aspects of the system are targeted, such a toy model can be very valuable.

It should also be noted that the ultimate aim of constructing the model is to utilize it to test more sophisticated control schemes. Some of the available schemes such as the optimal control theory require the system equations to estimate the whole flow field from limited measurement information and to predict the reaction to the control. Although a more accurate model would contribute to better prediction of the system, thus better performance of control, full simulation of the Navier–Stokes equations could hardly catch up with the real time experiment in most of the flow cases. In low dimensional problems such as the current problem and the thermal convection loop presented by Bau and Torrance [16], there is a better chance that a simple set of ordinary differential equations can be sufficient, even with fairy strong nonlinearity.

The outline of the paper is as follows. Section 2 describes the formulation of the toy model. In Section 3, for the toy model with absence of control, general features of nonlinear dynamics is shown. The calibration procedure is shown in Section 4, where the model is calibrated to the annular configuration. In Section 5, control problems with feedback control are analyzed for an ideal case and a limited case with local actuation. Results are compared to the experimental results and the cause of the limitation of the control is discussed. Finally concluding remarks are addressed in Section 5.