Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
European Journal of Mechanics B/Fluids 24 (2005) 338–352
EUROPEAN JOURNAL OF MECHANICS B/FLUIDS

Investigation of laminar flow in a helical pipe filled with a fluid saturated porous medium

L. Cheng, A.V. Kuznetsov *

Department of Mechanical and Aerospace Engineering, North Carolina State University, Campus Box 7910, Raleigh, NC 27695-7910, USA

Received 24 March 2004; received in revised form 18 August 2004; accepted 25 August 2004

Available online 18 October 2004

Abstract

Laminar flow in a helical pipe filled with a fluid saturated porous medium is investigated numerically. The analysis is based on a full momentum equation for the flow in porous media that accounts for the Brinkman and Forchheimer extensions of the Darcy law as well as for the flow inertia. Accounting for the flow inertia is shown to be important for predicting secondary flow in a helical pipe. The effects of the Darcy number, the Forchheimer coefficient as well as the curvature and torsion of the helical pipe on the axial flow velocity and secondary flow are investigated numerically.

Keywords: Helical pipe; Porous medium; Laminar flow; Orthogonal helical coordinates

1. Introduction

Flow in helical pipes has been a subject of intensive investigation. The major advantage of helical pipe flow is the occurrence of a secondary flow in planes normal to the main flow. Secondary flow increases heat and mass transfer efficiency compared to that in straight pipes. Dean [1] initially studied the flow in loosely coiled pipes and found the secondary flow with two symmetric vortices. Besides the Reynolds number, *Re*, another parameter, the Dean number, *Dn*, was later introduced to characterize the magnitude and the shape of the secondary flow. Germano [2,3] suggested an orthogonal helical coordinate system and used it to solve a laminar flow problem with a small ratio of torsion to curvature. He concluded that torsion has a second-order effect on the helical pipe flow. The Germano number, *Gn*, was introduced by Liu and Masliyah [4]. The Dean number given by $Dn = \varepsilon^{1/2} Re$ is a measure of the ratio of the square root of the product of inertial and centrifugal forces to the viscous forces. The Germano number given by $Gn = (\varepsilon\lambda) Re$ is a measure of the ratio of the centrifugal forces to the viscous forces (Liu and Masliyah [5]). Liu and Masliyah [4,5] performed a comprehensive analysis of fully-developed laminar Newtonian flows in helical pipes of constant circular cross-sections with a finite pitch and found that when the torsion is dominant, the flow in helical pipes approaches that in a straight pipe. When the torsion is small, the developing flow is oscillatory and the flow develops more quickly than in a straight pipe. Numerical studies have been conducted later to examine the effects of torsion and curvature on the fluid flow in helical pipes [6–11].

* Corresponding author. Tel.: +1-919-515-2365; fax: +1-919-515-7968.
E-mail address: avkuznet@eos.ncsu.edu (A.V. Kuznetsov).

0997-7546/$ – see front matter
doi:10.1016/j.euromechflu.2004.08.002

Nomenclature

a	pipe radius, m
C_F	Forchheimer coefficient
Da	Darcy number, K/a^2
Dn	Dean number, $\varepsilon^{1/2}Re$
g	gravity, $\mathrm{m/s^2}$
Gn	Germano number, $(\varepsilon\lambda)Re$
h_s	dimensionless scale factor
$\tilde{h}_r, \tilde{h}_s, \tilde{h}_\theta$	dimensional scale factors
K	permeability, $\mathrm{m^2}$
Re	Reynolds number, $\rho_f Ua/\mu$
r	dimensionless radial coordinate, $\tilde{r}/a$
$\tilde{r}$	radial coordinate, m
$\hat{\mathbf{r}}$	residual vector
s	dimensionless axial coordinate, $\tilde{s}/a$
$\tilde{s}$	axial coordinate, m
p	pitch, m
P	dimensionless pressure, $\widetilde{P}/\rho U^2$
$\widetilde{P}$	pressure, Pa
U	bulk velocity, defined in Eq. (2.9), m/s
$\tilde{\mathbf{v}}$	velocity vector, m/s
u_s, u_r, u_θ	dimensionless velocity components, $\tilde{u}_s/U$, $\tilde{u}_r/U$, $\tilde{u}_\theta/U$
$\tilde{u}_s, \tilde{u}_r, \tilde{u}_\theta$	velocity components, $\mathrm{m\,s^{-1}}$

Greek symbols

ε	dimensionless curvature, κa
θ	angle, defined in Fig. 1(b)
κ	curvature, $\mathrm{m^{-1}}$
λ	the ratio of torsion to curvature, τ/κ
μ	effective dynamic viscosity of the porous medium (assumed to the same as the fluid viscosity), $\mathrm{kg\,m^{-1}\,s^{-1}}$
ν	effective kinematic viscosity in a porous medium, $\mathrm{m^2\,s^{-1}}$
ξ	angle, defined in Eq. (2.10)
ρ_f	fluid density, $\mathrm{kg\,m^{-3}}$
τ	torsion, $\mathrm{m^{-1}}$
φ	porosity
ϕ	angle, defined in Eq. (2.3)

Subscripts

s	axial direction
r	radial direction
θ	circumferential direction

Sandeep [12] extended the analysis of helical pipe flow to non-Newtonian fluids; the numerical research was performed in a Cartesian coordinate system. Cheng and Kuznetsov [13] used the orthogonal helical coordinate system to study the effects of torsion and curvature on non-Newtonian fluid flow in helical pipes and compared the flow dynamics between Newtonian and non-Newtonian fluids.

Nield and Kuznetsov [14] presented a perturbation analysis and obtained an analytical expression for the Nusselt number in a helical pipe filled with a porous medium for the case when flow in a pipe is described by the Darcy law. Except for this paper, to the best of the authors' knowledge, nothing has been published on flows in helical pipes filled with a fluid saturated porous medium. The aim of the present paper is to fill this gap in the literature. Flow in helical pipes filled with a fluid saturated porous medium is relevant to a number of engineering and biological applications, such as the flow in a helical segment of clotted human coronary artery. This paper investigates laminar flow in a helical pipe filled with a porous medium. Since the secondary flow in a helical pipe becomes significant at relatively large flow velocity, it is insufficient to describe the drag that the porous medium imposes on a fluid by using just one linear (Darcy) term; at larger filtration velocities the surface drag due to friction becomes comparable with the form drag due to solid obstacles (Nield and Bejan [15]). Since the form drag due to solid obstacles is proportional to the square of the filtration velocity, to account for this effect an additional quadratic drag term is introduced into the momentum equation, which is called the Forchheimer term. In this paper, the most general form of a momentum equation for porous media, the Brinkman–Forchheimer-extended Darcy equation with inertia terms, is utilized; this equation is solved numerically in an orthogonal helical coordinate system suggested by Germano [2,3]. The geometry of a helical pipe is characterized by the curvature and torsion. In this study, the effects of the Darcy number, the Forchheimer coefficient, the curvature and torsion of the helical pipe on the axial flow velocity and secondary flow are investigated numerically. The investigation shows that increasing the Darcy number increases the distortion of the axial velocity profile and enhances the secondary flow. Increasing the Forchheimer coefficient decreases the axial velocity and the secondary flow. The dimensionless curvature of helical pipes affects both the axial velocity distribution and the secondary flow, but the ratio of torsion to curvature produces a noticeable effect only on the secondary flow.