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Shaping of sessile liquid metal drops using high-frequency magnetic fields

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Abstract

An analytical model for the electromagnetic shaping of sessile liquid metal drops in high-frequency magnetic fields is presented. We deal with both an infinitely long drop and a circular drop. In each case, the arrangement of drop and inductor is symmetric. Applying the skin depth approximation reduces the Lorentz forces induced in the liquid metal drop to a magnetic pressure on the drop surface. We neglect the coupling between drop contour and magnetic field distribution. In this case, the magnetic field can be calculated analytically applying the mirror-current method. Finally, we achieve an analytical solution of the static drop contours with the help of Green's functions. The theory is applied to three problems: (i) squeezing a drop while conserving its volume, (ii) drops with a fixed contact line, (iii) pumping up of drops. The results demonstrate the suitability of high-frequency magnetic fields for the shaping of liquid metals.

Keywords: Liquid metal; Free surface; Alternating magnetic field; Electromagnetic casting; Sessile drop

1. Introduction

Metallurgical applications increasingly involve magnetic fields. Via the Lorentz forces, induced in the electrically conducting liquid, they allow contactless heating and controlling of chemically aggressive, hot melts. Common applications include induction heating, electromagnetic stirring and pumping, and affecting liquid metal jets while pouring (Davidson [1]). Another application of magnetic fields is electromagnetic shaping. In this case, the induced Lorentz forces are used to control the free surface of liquid melts. Examples are cold-crucible technologies (Tanado [2]) and semi-levitation (Takeuchi [3]). Here, the idea is to prevent intense contact between the melt and crucible material. For example, Evans et al. [4] describe the production of a metal column which is slowly moving downward while the melt, held by electromagnetic forces, flows onto its top and solidifies. A similar procedure for electron beam evaporation is proposed by Kocourek et al. [5]. In this case, the solid metal column moves upward. On the top it is molten and then evaporated by an electron beam. A high-frequency magnetic field prevents the hot melt from draining.

Motivated by these potential applications, we investigate the behavior of liquid metal drops in high-frequency magnetic fields. Molokov and Reed [6] study free surface melt flows in static magnetic fields theoretically using an asymptotical method for strong magnetic fields. They consider a drop-shaped rivulet flowing down an inclined plane and submitted to a strong

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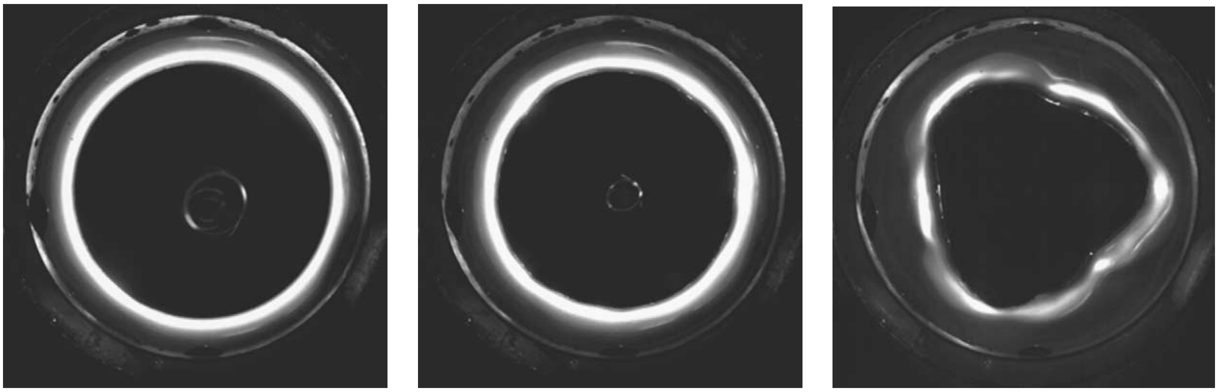

(a) (b) (c)

Fig. 1. Experimental observation of a liquid metal drop affected by a high-frequency magnetic field. (a) Static drop shape in the absence of the magnetic field ($I = 0$), (b) static drop shape in the presence of a magnetic field at $I = 80$ A and $f = 20$ KHz, (c) dynamic drop shape in the presence of a magnetic field at $I = 150$ A and $f = 20$ KHz. The drop slowly oscillates with a frequency in the range of $f = 2$–3 Hz and a mode number $m = 3$.

magnetic field perpendicular to flow direction. The magnetic field is directed normal as well as tangential to the plane. In their theory, the contact angle is assumed to be fixed as no experimental data is available describing the contact angle dynamics in magnetic fields. For magnetic fields normal to the plane, the flow inside the rivulet is determined by an equilibrium of Lorentz forces and gravity. At the bottom, a thin Hartmann boundary layer is built up which reduces the flow velocity. In the case of a horizontal magnetic field, gravity is balanced by friction forces. The flow velocity is independent of the magnetic field strength. Fautrelle et al. [7] studied the dynamic behavior of liquid metal drops in low-frequency magnetic fields. The applied frequencies are of a few hertz, corresponding to the eigenfrequencies of a spherical drop (Landau and Lifschitz [8]). The analysis shows that resonance effects occur, i.e. the frequency of the generated surface wave is a multiple of the exciting frequency. In the present paper we focus on electromagnetic shaping of liquid metal drops using fields of high frequency of some kilohertz. Due to the skin effect (Jackson [9]), the induced Lorentz forces then act like an additional pressure on the free surface of the drop. For example, this effect is also used by Riahi and Walker [10] as well as Lie et al. [11] who tackle the problem of float zone shaping in silicon crystal growth using radio frequency of about $f = 3$ MHz.

In our case, accompanying experiments [12] indicate the drop dynamics. Figs. 1(a)–(c) show contours of a liquid drop of the low-melting metal Galinstan, photographed from above. The drop volume remains $V = 7$ ml in all three pictures. Fig. 1(a) shows the circular contour of the drop in the absence of a magnetic field. Fig. 1(b) shows the drop contour when submitted to a high frequency magnetic field. The field is generated by a circular inductor fed by an alternating current of $I = 80$ A and $f = 20$ kHz. Although a weak undulating is observed, the drop retains its circular shape. Moreover, the circumference is reduced. However, on increasing the inductor current beyond a critical value, the circular shape becomes unstable, see Fig. 1(c). For $I = 150$ A and $f = 20$ kHz the drop oscillates slowly (2–3 Hz) around its equilibrium shape with a mode number depending on drop volume. The analytical model described in this paper predicts the static deformation of liquid metal drops in high-frequency magnetic fields. For simplification, the analysis is restricted to flat drops with fixed contact angle. We consider infinitely long drops as well as circular drops. In both cases, we focus on a symmetrical arrangement to obtain a two-dimensional problem.

The paper is organized as follows. In Section 2 we present the analytical model. We derive an inhomogeneous differential equation that describes the static shape of the drop. Restricting the analysis to flat drops, we achieve a solution using Green's function theory. In Section 3 we show the main results of our analysis. Three types of shaping problems are discussed: (i) squeezing of a drop, (ii) the static equilibrium shapes of drops with fixed contact line, (iii) the pumping up of a drop in an applied field. Section 4 provides a short summary.

2. Analytical model

2.1. Description of the arrangement

Fig. 2 shows a sketch of the long drop (Fig. 2(a)) as well as the circular drop (Fig. 2(b)). While the long drop is described in a Cartesian x–z-plane, cylindrical coordinates r–z are used for the circular drop. The drop is placed on an electrically conducting ground plate at $z = 0$. The drop contour is given by $h(x)$ for the long drop and $h(r)$ for the circular drop. The contour touches