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Surface waves created by low-frequency magnetic fields

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Abstract

This paper analyses the effects of a low frequency A.C. magnetic field on the free surface of a liquid metal. The action of the vertical and uniform magnetic field is twofold. First it creates forced standing surface waves which generally exhibit symmetry related to that of the container; second it triggers non-symmetric free surface instabilities superimposed on the forced regime. A previous paper considered the case of a circular cylindrical tank where axisymmetric forced standing waves caused an electric current perturbation which then excited non-axisymmetric waves at a critical A.C. field intensity. Nonlinear interaction between the symmetric and non-symmetric modes was not taken into account. The present work treats the problem from a more general standpoint. Equilibrium perturbations are developed systematically to order $\mathfrak{N}^2$ (where $\mathfrak{N}$ is the magnetic interaction parameter) and at this level of approximation we also need to consider nonlinear mode interactions and electromagnetic damping. The theory applies to tanks of arbitrary shape and the $O(\mathfrak{N})$ irrotational motion may be described by the torsion function for the particular pool cross-section. For circular and annular tanks we then derive a system of coupled Mathieu–Hill equations for the time-development of non-symmetric surface modes. Two main types of parametric resonance are predicted, namely the single or combination mode, and the particular type observed may depend on the geometry of the tank. Results of the stability analysis are confirmed by experimental work carried out in mercury pools.

Keywords: MHD; Liquid metal; Alternating fields; Parametric resonance

1. Introduction

A.C. magnetic fields are commonly used in various metallurgical devices for heating and stirring liquid metals. Typical frequencies range from 50 Hz to 10^5 kHz. In that frequency range only the mean part of the Lorentz forces plays a significant role, the liquid metal inertia being too large to respond to the oscillating part of the Lorentz forces. The very low frequency range, i.e. from about 1 to 10 Hz, has been quite recently explored. In this case, it has been shown both experimentally and theoretically that the oscillating part of the electromagnetic forces could generate waves on the free surface of a liquid metal [1,2] (referred to hereafter as GFS). This phenomenon occurs when the magnetic field and the first few normal wave modes have frequencies of the same order. Mercury experiments in a cylindrical tank [1] have revealed two principal types of waves. For weak magnetic fields, axisymmetric quasi-steady waves appear on the free surface. These are directly forced by the alternating part of the electromagnetic forces. It was also found that above a critical magnetic field intensity symmetry-breaking occurred and azimuthal modes developed. Their oscillation frequency in the vicinity of the transition was half that of the electromagnetic force.

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Low-frequency fields are currently used in a number of industrial processes. For example a low frequency pulsed A.C. magnetic field in the continuous casting of steel is able to replace mechanical mold vibrations [3]. Contactless free surface stirring may also be used to accelerate the kinetics of mass transfer in slag-metal refining reactions [4].

A theoretical linear stability analysis has been performed by GFS in cylindrical geometry. This showed the existence of a basic state consisting of forced axisymmetric waves. Moreover their simplified stability analysis showed that non-symmetric modes were governed by a system of generalised Mathieu–Hill equations. Thus the appearance of unstable non-symmetric modes could be attributed to parametric resonance. In GFS parametric forcing arises from the electromagnetic force perturbation due to azimuthal surface waves. Any non-symmetric free surface perturbation alters the flow path of the basic axisymmetric induced current and the resulting electromagnetic force perturbation destabilises the free surface. However, in their analysis GFS ignored the influence of both the axisymmetric wave motion as well as the electric current created by the fluid motion across the magnetic field lines. It turns out that nonlinear interactions with the axisymmetric modes are of the same order of magnitude as the electromagnetic force perturbations. We show later that both are of $O(\mathfrak{N}^2)$, and the former effect is likely to have a significant effect on stability. Furthermore, the magnetic field strength required to trigger the instability is quite high, so we might expect electromagnetic damping to be significant.

The present work is a continuation of previous analysis by GFS, but from a more general standpoint. We use an inner-product method to formulate evolution equations for the wave mode amplitudes, performing a systematic expansion to order $\mathfrak{N}^3$, where $\mathfrak{N}$ is the magnetic interaction parameter. This expansion shows it necessary to consider the two effects mentioned above, omitted in the GFS analysis:

(i) nonlinear interactions between the forced-wave regime and non-symmetric modes;
(ii) electric currents induced by the fluid motion across the magnetic field lines.

The inclusion of these two effects produces results which are significantly different from those of GFS. Field strength is measured by $\mathfrak{N}$ and the critical value of $\mathfrak{N}$ necessary to trigger the growth of a parametrically excited mode is denoted by $\mathfrak{N}_C$. Since effect(i) is destabilising we obtain values of $\mathfrak{N}_C$ which are consistently smaller the corresponding GFS values — in other words we find parametric instabilities much easier to excite. This effect is particularly important when the field frequency is nearly half that of a forced wave; then the forced-wave amplitude becomes very large and non-symmetric modes are triggered by a relatively weak field. On the other hand, magnetic damping, effect (ii), suppresses many unstable modes found by GFS. Indeed one complete class of instabilities entirely disappears.

In section two we establish the equations governing forced surface waves — the first-order basic state. Our method of analysis is formulated for tanks of arbitrary cross-section, and as well as the circular tank we consider rectangular and annular geometries. Then in section three we continue the expansion of the evolution equations to $O(\mathfrak{N})^2$. An important advantage of the inner product method is that the order $\mathfrak{N}^2$ rotational flow does not have to be found explicitly. We then use the nonlinear evolution equation to derive a linear equation describing the growth of non-symmetric modes in cylindrical or annular tanks. Section four is devoted to analysis of results and comparisons between theory and experiment. Conclusions are summarised in section five.

2. First-order analysis

We consider a tank with vertical walls and an arbitrary (but uniform) horizontal cross-section. Our method is most clearly formulated if we make no particular assumptions about the tank cross-section, but later we specialise to cylindrical and annular geometries. The tank contains fluid of density ρ and electrical conductivity σ to a depth h, and a uniform vertical alternating magnetic field

$$\boldsymbol{B} = B_0 \sin(\omega t)\hat{z} \tag{1}$$

is applied (Fig. 1). We assume the fluid incompressible so ρ is constant and

$$\nabla \cdot \boldsymbol{v} = 0,$$

where $\boldsymbol{v}$ is fluid velocity. The time-dependent field induces an electric current in the fluid which interacts with the field to produce a Lorentz force, which stirs the fluid and creates a pattern of standing waves on the free surface.

The first-order analysis is dedicated to the determination of the forced regime. We shall make two approximations throughout: first that the magnetic diffusion time $L^2\mu_0\sigma$ (where L is a typical length scale and μ_0 the permeability of free space) is much larger than an oscillation period — i.e.

$$\omega L^2 \mu_0 \sigma \ll 1, \quad \text{or} \quad R_m^* \ll 1, \tag{2}$$