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Inverse determination of a heat source from a solute concentration generation model in porous medium ☆

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Abstract

The conjugate gradient method with adjoint equations is applied to the natural convection problem in a porous medium for the determination of an unknown heat source which is dependent on a solute concentration generation rate. The direct, sensitivity and adjoint equations are given for a Boussinesq fluid, over an arbitrary domain in two dimensions. Solutions by control volumes are presented for a square enclosure under known temperature and concentration boundary conditions, assuming a source term proportional to the vertical average generation rate of a solute concentration governed by a Monod model. Reasonably accurate solutions are obtained at least up to $Ra=10^5$.

Keywords: Inverse convection; Porous medium; Biochemical heat source

1. Introduction

Over the last few years, a tendency has emerged regarding the environmental management of organic waste. Indeed, several options are now available to transform it into products at the same time less harmful to the environment and reusable to some extent. In this context, it is important to develop tools to refine our analysis of the physical, chemical, and biological phenomena involved. In the area of sanitary landfill for instance, El-Fadel et al. [1] developed a complete numerical model to describe the generation and transport of gases and heat. Biokinetic equations are included in their model to take into

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account the microbial activity within the landfill. Also, the experiments with pulp and paper waste of Larsen and McCartney [2] and others recently confirmed that the composting process is essentially controlled by internal heat generation from microbial oxidation, with a strong relationship between microbial activity and chemical solute concentration ratios.

Any effective control of these processes requires some level of monitoring of either temperature, concentration, and/or heat generation distribution inside these systems. Anaerobic decomposition or composting may be tough of as a double-diffusive convection problem in a porous medium with a heat source. Few studies were ever undertaken on this topic. Among these, Chamkha [3] recently treated the double-diffusive problem for a gas mixture in a rectangular enclosure with heat and mass gradients applied on the vertical walls and a heat source depending linearly upon temperature.

The inverse problem approach, which is designed to estimate boundary conditions or thermophysical properties of a system where direct measurements are impracticable, offers an interesting way to determine the heat source distributions within the reactor through the use of remote temperature measurements taken within the system itself.

Some information are available in the literature concerning inverse heat transfer problems involving convection in general and natural convection in particular. The sequential function specification method [4] was used by Moutsoglou [5] to study inverse laminar natural convection in a fluid medium in a vertical channel. Park and Chung [6] and Park and Jung [7] solved the inverse natural convection problem using different approaches for an unknown time-dependent heat source inside a square cavity. Prud'homme and Nguyen [8] and Zabaras and Yang [9] used the conjugate gradient method with adjoint equations to study the inverse natural convection problem. In a porous medium, Prud'homme and Jasmin [10] adapted the conjugate gradient method to solve the inverse natural convection problem with mass diffusion, for a heat source which is a function of the solute concentration.

2. Problem definition

The conjugate method will be used to solve the inverse natural convection problem in a porous medium in two dimensions. The domain geometry and boundary conditions are summarized in Fig. 1. Our objective is to determine the unknown heat source Q over the time interval $0 \le t \le t_f$, from the

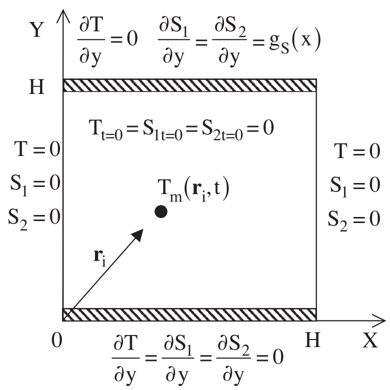

Fig. 1. Geometry and boundary conditions.