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Stability of gravitactic micro-organisms in a fluid-saturated porous medium☆

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Abstract

A study is made of the stability of motile suspensions in a horizontal porous layer. The micro-organisms are assumed to have a gravitactic behaviour, swimming randomly, but on the average upward with a constant velocity V_c. The resulting equilibrium state is potentially unstable as a denser, layer of micro-organisms is formed on top of a lighter one. The basic mechanism is analogous to that of Bénard convection in a fluid layer heated from below. The fluid flow is governed by the Darcy equation while the conservation of micro-organisms is described by a diffusion–convection equation similar to the conservation of energy. The problem depends on two parameters, namely the Rayleigh number and the swimming velocity V_c. The present paper is focused on the stability of the equilibrium diffusive state. The stability diagram and the critical conditions for the onset of convection are obtained for a wide range of swimming velocity. It is found that if V_c is very small, the critical wavenumber is zero, corresponding to a very long cell (parallel flow), but as V_c is increased, the critical wavenumber also increases, corresponding to narrower flow patterns.

Keywords: Bioconvention; Gravitactic micro-organisms; Porous medium; Stability

1. Introduction

Bioconvection is the flow-pattern formation in suspensions of motile micro-organisms whose density is greater than that of the fluid [1,2]. These patterns appear because bacterial cells or motile

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algae may swim towards the surface of the fluid, thus creating a heavy layer on top of a lighter one. If the top layer is sufficiently heavy, the situation will become unstable and the micro-organisms will fall abruptly into a collection of plumes. From a hydrodynamic point of view, the up-swimming micro-organisms and the fluid form a mixture with a non-uniform concentration, giving rise to a buoyancy force as in the case of a fluid layer heated from below. This is similar to the phenomenon of Bénard convection, hence the name *bioconvection* coined by Platt in 1961 [4]. For a detailed description of the hydrodynamics of swimming cells, we refer to the papers of Pedley and Kessler [2,3] and Ghorai and Hill [5,6].

A model of pattern formation has been developed by Childress, Levandowsky, and Spiegel [7] for gravitactic micro-organisms, based on the Navier–Stokes equation for fluid flow and the diffusion–convection equation for the conservation of motile micro-organisms. They first derived the *equilibrium state* resulting from the upward swimming and the downward diffusion of the motile organisms in a quiescent fluid. They then determined analytically the critical Rayleigh number for the onset of convection as well as the preferred wavenumber and growth rates. A review of this subject was given by Pedley and Kessler [2]. Fujita and Watanabe [8] presented a numerical study of bioconvection based on the mathematical model of Childress et al. [7] to show the transition from a static condition to periodic oscillation as the Rayleigh number is increased. It was found that the bioconvection flow could become chaotic via a single-frequency oscillatory behaviour to a sequence of period-doubling bifurcations. A study of the preferred wavenumber in a suspension of gravitactic micro-organisms in a 2*D* rectangular cavity was made by Harashima, Watanabe, and Fujishiro who carried out numerical experiments to show how the number of falling fingers is selected at each stage of evolution. They found that the "evolution of the system proceeds in the direction of intensifying the downward advection of micro-organisms and reducing the total potential energy of the system" [9].

Recently, Kuznetsov, and Jiang presented a study of gravitatic bioconvection in a porous medium. The Darcy equation for fluid flow and the diffusion–convection equation for the concentration of micro-organisms were numerically solved to obtain the flow and concentration fields in a square cavity in terms of the permeability of the porous medium. They concluded that "there exists a critical value of permeability for the onset of convection and after numerous computations, it was determined that that the critical value of permeability is approximately $4\times10^{-7}\mathrm{m}^2$. If permeability is smaller than this value, then no convection develops. This, in turn, causes the cells to accumulate in the top layer and stay there" [10].

In the present paper, we consider the problem of bioconvection in a porous layer saturated with a suspension of gravitactic micro-organisms. The onset of convection will be determined by the linear stability theory. The stability diagrams, the critical Rayleigh numbers and the critical wavenumbers will be presented for a wide range of gravitactic velocity of the motile organisms.

2. Mathematical formulation

2.1. Governing equations

Consider a 2D porous medium containing a concentration of gravitactic micro-organisms swimming with a constant upward velocity $\vec{V}_c^*$ as shown in Fig. 1. The suspension is assumed to be incompressible