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Friction drag resulting from the simultaneous imposed motions of a freestream and its bounding surface

J.P. Abraham [a,*], E.M. Sparrow [b]

[a] Department of Mechanical Engineering, University of St. Thomas, 2115 Summit Avenue, St. Paul, MN 55105-1079, USA
[b] Department of Mechanical Engineering, University of Minnesota, Minneapolis, MN 55455-0111, USA

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Abstract

The problem of laminar fluid flow which results from the simultaneous motions of a freestream and its bounding surface in the same direction has been investigated numerically. A special focus was to establish thresholds demarking the degree of interaction between the two imposed motions. For the case in which the freestream velocity, U_∞, is greater than the surface velocity, U_S, it was found that surface velocities as great as $0.183U_\infty$ could be tolerated without causing an appreciable effect on the drag force. On the other hand, in the case where the surface velocity exceeds the freestream velocity, the drag is significantly affected by freestream velocities $\geqslant 0.0686U_S$.

The validity of the so-called *relative-velocity model* was investigated. In that model, the relative velocity between the two media is used in conjunction with the drag-force formula for the dominant of the two motions acting alone. The results of exact solutions demonstrate that this model is flawed and underpredicts the drag force. Finally, a new model, termed the *similarity-based, relative-velocity model* was devised by which exact similarity solutions were obtained. The drag results provided by this model may be considered to be the definitive information for application.

Keywords: Moving surface; Moving fluid; Drag forces; Laminar flow; Exact similarity solutions; Web processing

1. Introduction

In the processing of web-like materials such as polymer sheets, paper, linoleum, or roofing shingles, one of the commonly encountered approaches is to cure and/or dry the moving web by the use of a parallel gas flow. A schematic diagram of the process setup is presented in Fig. 1. As seen there, the material to be processed is supplied from a roll situated at the upstream end of the processing station. The processed product is collected by a take-up roll located at the downstream end of the station. The gas flow is supplied by means of a converging nozzle, the degree of convergence being such that the boundary layer thickness of the gas flow at the nozzle exit is negligible. The flow emerges from the nozzle as a straight, parallel freestream with uniform velocity. The relative motion between the unrolling web and the edge of the nozzle is accommodated by brushes. A side-view of the physical situation illustrated in Fig. 1 is displayed in Fig. 2.

In the textbook literature (Bejan, 1993; Cengel, 2003; Incropera and DeWitt, 2002; Kaviany, 2002; Kreith and Bohn, 2001; Rohsenow and Choi, 1961; Suryanarayana, 1995), it appears that no distinction has been drawn between the fluid flow adjacent to a moving web such as that pictured in Figs. 1 and 2 and the classical stationary flat-plate boundary layer problem, often termed the Blasius problem (Blasius, 1908). The conclusion that can be drawn from the aforementioned textbooks is that those authors have assumed that when a moving fluid passes

* Corresponding author. Tel.: +1 651 962 5766/5750; fax: +1 651 692 6419.
E-mail address: jpabraham@stthomas.edu (J.P. Abraham).

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Nomenclature

$f(\xi)$	similarity function based on the relative velocity between the fluid and the bounding surface
F	drag force
F_{exact}	drag force based on a similarity solution
$F^{U_{rel}^{I}}$	drag force based on relative-velocity model for $U_\infty > U_S$
$F^{U_{rel}^{II}}$	drag force based on relative-velocity model for $U_S > U_\infty$
L	streamwise length of the bounding surface
u, v	velocity components in the x and y directions, respectively
U_∞	freestream velocity
U_S	velocity of the moving surface
U_{rel}	relative velocity between the moving surface and the freestream
W	width of bounding surface normal to the direction of motion
x, y	horizontal and vertical coordinates

Greeks

α	velocity ratio, U_∞/U_S
μ	dynamic viscosity of fluid
ν	kinematic viscosity of fluid
ψ	streamfunction
ξ	similarity variable based on the relative velocity, U_{rel}

Superscripts

I	corresponds to the case for which $U_\infty > U_S$
II	corresponds to the case for which $U_S > U_\infty$

over a stationary solid and/or when a moving solid passes through a stationary fluid, it is only the *relative velocity* between the surface and the fluid that determines the fluid flow and heat transfer characteristics. This model will hereafter be termed the *relative-velocity model*. For the determination of the drag force, this model uses the magnitude of the relative velocity in conjunction with the drag formula for the case in which only one of the participating media is in motion.

The problem of a surface moving through a stationary fluid was first analyzed by Sakiadis (1961) and experimentally investigated by Tsou et al. (1967). Subsequently, the situation in which both the surface and the fluid are in motion in the same direction has been analyzed by Abdelhafez (1985) and Chen (1998, 1999). Although these papers present a variety of numerical results, the critical issue of the relative velocity as discussed in the preceding paragraph was not elucidated. It is interesting that the results were presented in terms of what would appear to be a relative velocity, but is, in fact, a velocity ratio.

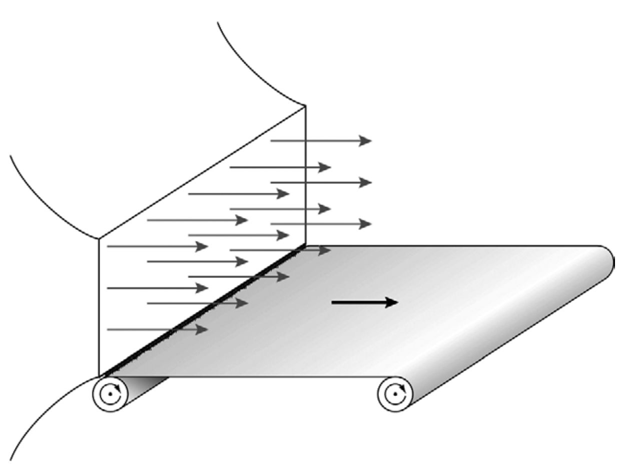

Fig. 1. Processing station consisting of a moving web situated in a parallel gas flow.

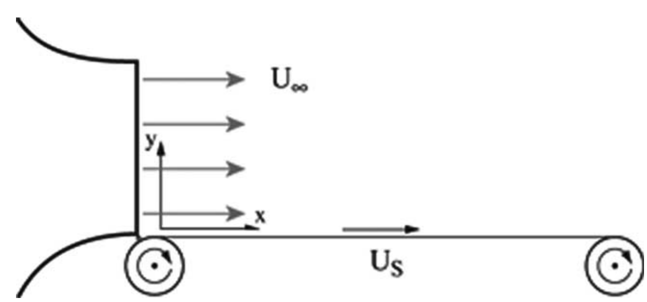

Fig. 2. Sideview of the processing station depicted in Fig. 1.

There are three foci that will be addressed in the present work. One of these is a complete numerical elucidation of the role of the relative velocity and an accounting of the errors that would be made if the relative-velocity model were used. A second focus addresses the issue of when it is permissible to neglect the slower of the participating motions without incurring a significant error. Finally, a new similarity solution based on the proper use of the relative velocity is obtained which further demonstrates the inadequacy of the relative-velocity model.

2. Drag-force results

The analysis of the fluid flow field resulting from the simultaneous motions of a moving freestream and a moving bounding surface lends itself to analysis via the method of similarity solutions (Abdelhafez, 1985). This is a very useful tool because it enables the problem,