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International Journal of Heat and Mass Transfer 48 (2005) 1096–1106
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Sudden or smooth transitions in porous media natural convection

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Received 29 March 2004; received in revised form 15 September 2004
Available online 8 December 2004

Abstract

In porous media isothermal flow a transition from the Darcy regime, via an inertia dominated regime, towards turbulence is anticipated. In porous medium natural convection the transition to turbulence follows a different route. The first transition from a motionless-conduction regime to steady natural convection is followed by a direct second transition to a non-steady (time dependent) and non-periodic regime (referred to as weak turbulent), prior to the amplitude of the convection reaching such large values as to involve inertial, non-Darcy effects. The latter is due to an additional non-linear interaction that appears in natural convection as a result of the coupling between the equations governing the fluid flow and the energy equation. The present paper deals with identifying whether the transitions are sudden or possibly smooth. The latter is accomplished by using a truncated Galerkin representation of the natural convection problem in a porous layer heated from below (an extended Darcy model) leading to the familiar Lorenz equations for the evolution of the convection amplitudes with time. Two different formulations (named the "original" and the "modified" systems) are being used in an anticipation to obtaining a smooth transition in the form of an imperfect bifurcation from the "modified" system formulation. The results show that the transition remains sudden and the accuracy of the "modified" system results is being tested in comparison with the "original" system showing a sufficiently high degree of accuracy.

Keywords: Weak turbulence; Natural convection; Porous media; Chaos

1. Introduction

In porous medium natural convection the transition to turbulence follows the following route. The first transition is from a motionless-conduction regime to steady natural convection. This is followed by a direct second transition to a non-steady (time dependent) and non-periodic regime (referred to as weak turbulent), prior to the amplitude of the convection reaching such large values as to involve inertial, non-Darcy effects. The latter is due to an additional non-linear interaction that appears in natural convection as a result of the coupling between the equations governing the fluid flow and the energy equation.

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0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2004.09.039

Nomenclature

Da	Darcy number, defined by k_*/H_*^2
H_*	the height of the layer
H	the front aspect ratio of the porous layer, equals H_*/L_*
k_*	permeability of the porous domain
L_*	the length of the porous layer
L	reciprocal of the front aspect ratio, equals $1/H = L_*/H_*$
M_f	ratio of the fluid and the porous domain heat capacities
p	reduced pressure (dimensionless).
Pr	Prandtl number, equals ν_*/α_{e*}
Ra	porous media gravity related Rayleigh number, equals $\beta_*\Delta T_C g_* H_* k_* M_f/\alpha_{e*}\nu_*$
Ra_0	critical Rayleigh number value for loss of stability of the steady convection solution
R	scaled Rayleigh number, equals $Ra/4\pi^2$
R_0	critical value of R for the loss of linear stability of the steady convection solution
r	absolute value of the complex amplitude
r_0	initial condition of r
t	time
T	dimensionless temperature, equals $(T_* - T_C)/(T_H - T_C)$.
T_C	coldest wall temperature
T_H	hottest wall temperature
u	horizontal x component of the filtration velocity
v	horizontal y component of the filtration velocity
w	vertical component of the filtration velocity
x	horizontal length co-ordinate
X	amplitude of convection flow defined in Eq. (6)
y	horizontal width co-ordinate
Y	amplitude of convection temperature defined in Eq. (7)
z	vertical co-ordinate
Z	amplitude of convection defined in Eq. (7)

Greek symbols

α	a parameter related to the time derivative term in Darcy's equation
α_{e_*}	effective thermal diffusivity
β_*	thermal expansion coefficient
ε	asymptotic expansion parameter, see text following Eq. (17)
ϕ	porosity
χ	dimensionless group, equals $\phi Pr/Da$
ν_*	fluid's kinematic viscosity
μ_*	fluid's dynamic viscosity
ψ	stream function
ΔT_C	characteristic temperature difference
τ	long time scale
θ	the phase of the complex amplitude

Subscripts

*	dimensional values
t	transitional values
cr	critical values

Superscript

*	complex conjugate

Modeling the weak turbulent regime for natural convection in a fluid saturated porous layer is known to be extremely sensitive to initial conditions. The question of compatibility of the initial conditions between the computational and analytical (weak non-linear) solutions arises in particular in connection with the prediction of the transition point. Vadasz and Olek [1,2] demonstrated by using a computational method of solution (Adomian's decomposition method [3–5]) that the transition from steady to chaotic (weak-turbulent) convection in porous media can be recovered from a truncated Galerkin approximation which yields a system that is equivalent to the familiar Lorenz equations (Lorenz [6], and Sparrow [7]). In particular it was noticed that the transition to chaos occurs at a particular sub-critical value of Rayleigh number. Here, the term "sub-critical" is used in the context of the transition from steady convection to a non-periodic state, typically referred to as chaotic, and the critical value of the Rayleigh number is the value at which this transition to chaos is predicted by the linear stability analysis of the convective steady state solutions. The problem that the linear stability analysis reveals that the transition occurs at Rayleigh number values substantially smaller than those obtained by accurate numerical solutions (or by experimental results of an equivalent system that is governed by the same set of Lorenz equations, Yuen and Bau [8] and Wang et al. [9]) and in some cases at values smaller by 50% was addressed by Vadasz [10]. The latter showed that an analytical, weak non-linear, method of solution to this problem, can provide accurate transition values via a correction to the linear stability results. In addition, Vadasz [10] revealed a mechanism for the well known Hysteresis phenomenon in the transition from steady to chaotic convection and backwards. The transition from the steady to the chaotic solution occurs via a subcritical Hopf bifurcation (see Sparrow [7], Yuen and Bau [8] and Wang et al. [9], Vadasz [10,11]) and is asso-