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The onset of Lapwood–Brinkman convection using a thermal non-equilibrium model

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Abstract

The stability of a horizontal fluid saturated sparsely packed porous layer heated from below and cooled form above when the solid and fluid phases are not in local thermal equilibrium is examined analytically. The Lapwood–Brinkman model is used for the momentum equation and a two-field model is used for energy equation each representing the solid and fluid phases separately. Although the inertia term is included in the general formulation, it does not affect the stability condition since the basic state is motionless. The linear stability theory is employed to obtain the condition for the onset of convection. The effect of thermal non-equilibrium on the onset of convection is discussed. It is shown that the results of Darcy model for the non-equilibrium case can be recovered in the limit as Darcy number $Da \rightarrow 0$. Asymptotic analysis for both small and large values of the inter phase heat transfer coefficient H is also presented. An excellent agreement is found between the exact solutions and asymptotic solutions when H is very small.

Keywords: Convection; Thermal non-equilibrium; Brinkman model; Porous medium

1. Introduction

The problem of convective instability of horizontal porous layer subject to an adverse temperature gradient has been investigated extensively by several authors in the past [1–5]. It is important to note that all the above-mentioned studies are based on the Darcy model. However, it is now realized that the Darcy model is applicable only under special circumstances and a generalized model for the accurate prediction of convection in a porous media must include Brinkman viscous term and Forchheimer inertia term. During the last decade, there has been a great upsurge of interest in determining the effects of extensions to Darcy's law since many practical applications involve media for which Darcy's law is inadequate. Some of the early works to deal with these extension are by Rudraiah et al. [6], Georgiadis and Catton [7], and Kladias and Prasad [8] who used the Darcy–Brinkman (DB) model and Darcy–Brinkman–Forchheimer (DBF) model for studying Benard convection in porous media. Many more works are available on the non-Darcy–Benard convection in a porous medium. The growing volume of work devoted to this area is well documented by the most recent reviews of Nield and Bejan [9] and Ingham and Pop [10].

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Nomenclature

c	specific heat
d	height of the porous layer
Da	Darcy number, $\frac{\mu_e K}{\mu_f d^2}$
$\mathbf{g}$	gravitational acceleration
h	inter phase heat transfer coefficient
H	non-dimensional inter phase heat transfer coefficient, $\frac{hd^2}{\varepsilon k_f}$
k	horizontal wave number
k_f, k_s	thermal conductivities of fluid phase and solid phase respectively
K	permeability of the porous medium
Pr	Prandtl number, $\frac{\mu_e(\rho c)_f}{\rho_0 k_f}$
p	pressure
$\mathbf{q}$	velocity vector, (u, v)
Ra	Rayleigh number, $\frac{\rho_0 g\beta(T_l - T_u)Kd}{\varepsilon\mu_f\kappa_f}$
T	temperature
t	time
(x, y)	space coordinates

Greek symbols

α	diffusivity ratio
β	coefficient of thermal expansion
γ	porosity-modified conductivity ratio, $\frac{\varepsilon k_f}{(1-\varepsilon)k_s}$
ε	porosity
κ	thermal diffusivity
μ_e	effective viscosity
μ_f	fluid viscosity
ρ_f	fluid density
ψ	non-dimensional stream function
θ	non-dimensional temperature of the fluid phase
ϕ	non-dimensional temperature of the solid phase
∇^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

Subscripts/superscripts

f	fluid
l	lower
s	solid
u	upper
*	non-dimensional
o	reference

Most of the works on convective heat transfer in porous media have mainly been investigated under the assumption that the fluid and the porous medium are everywhere in local thermodynamic equilibrium (LTE), although in many practical applications the solid and the fluid phases are not in thermal equilibrium. Nield and Bejan [9] have discussed a two-field model for energy equation. Instead of having a single energy equation, which describes the common temperature of the saturated media, two equations are used for fluid and solid phase separately. In the two-field model, the energy equations are coupled by the terms, which account for the heat lost to or gained from the other phase. Rees and co-workers [11–14] in a series of studies have investigated the non-equilibrium effect on free convective flows in porous media using Darcy model. The review by Kuznetsov [15] gives a detailed information about the works on thermal non-equilibrium effects.

In this paper we study the onset of convection in a sparsely packed porous layer heated from below with emphasis on how the condition for the onset of convection is modified when the solid and fluid phase are not in local thermal equilibrium (non-LTE). As discussed by Banu and Rees [12] when non-equilibrium effects are included in the problem the linear analysis is modified and it is still possible to proceed analytically to find the condition for the onset of convection. We have also carried out the asymptotic analysis for very small and very large values of the inter phase heat transfer coefficient. This work is more general than that of Banu and Rees [12] in the sense that we recover their results in the limit as the Darcy number Da tends to zero.

2. Mathematical formulation

We consider a Boussinesq fluid saturated porous layer of depth d, which is heated from below and cooled from above. A Cartesian coordinate system is chosen with the origin on the lower boundary and y-axis vertically upward. The lower surface is held at temperature T_l, while the upper surface is at T_u. We assume that the solid and fluid phases of the medium are not in local thermal equilibrium and use a two-field model for temperatures. It is assumed that at the bounding surfaces the solid and fluid phases have identical temperatures. The Lapwood–Brinkman model is employed for the momentum equation. The basic governing equations are (see [9])

$$\nabla \cdot \mathbf{q} = 0 \tag{1}$$

$$\rho_f\left[\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon^2}\mathbf{q}\cdot\nabla\mathbf{q}\right] = -\nabla p - \frac{\mu_f}{K}\mathbf{q} + \mu_e\nabla^2\mathbf{q} + \rho_f\mathbf{g} \tag{2}$$

$$\varepsilon(\rho c)_f\frac{\partial T_f}{\partial t} + (\rho c)_f\mathbf{q}\cdot\nabla T_f = \varepsilon k_f\nabla^2 T_f + h(T_s - T_f) \tag{3}$$

$$(1-\varepsilon)(\rho c)_s\frac{\partial T_s}{\partial t} = (1-\varepsilon)k_s\nabla^2 T_s - h(T_s - T_f) \tag{4}$$