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Technical Note

Dimensional analysis for Gorter–Mellink counter flow convection in pressurized superfluid helium

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Abstract

Dimensional analysis of the heat transport through ducts filled with saturated He II is extended to pressurized conditions up to 20 bar. Simplified models are also presented for the ease of first order estimation of the GM-transport heat flux density, the temperature gradient and the limiting heat flux densities by relating calculations to reduced thermo physical properties, with respect to the properties at the lambda point. The data available show good support for the dimensionless GM-equation in pressurized He II within data scatter and thermo physical property uncertainty.

Keywords: Superfluid; Superconductor; Helium II; Low temperature; Heat transfer

1. Introduction

Superfluid liquid He II (He^4) is considered as a very unique and efficient coolant for high performance superconducting systems [1,2]. Much work has been done to understand the heat transport properties of He II where a large body of knowledge of its behavior at low temperatures and pressures are available. However, there are only a few comprehensive data sets available in suitable form in a particular application area. Therefore, the purpose of the present work is to present a simple set of functions which permit a quick selection of various system options. Empirical formulae are employed to relate the thermo physical properties to the reduced properties relative to the properties at lambda points. This leads to surprisingly simple functions which ease the calculation of the heat flux density at a given temperature and pressure. Experimental data are compared favorable with the dimensional analysis.

2. Laminar transport in zero net mass flow ($\Delta T \sim 0$)

At very low temperature difference, between the temperature of the heated end and the temperature of bath, the normal fluid flow in tubes is described by the Hagen-Poiseuille equations. One can generalize the laminar transport of He II with the following equation

$$N_q = N_{\nabla T} \tag{1}$$

where $N_q = \rho \upsilon_n L_c/\eta_n = qL_c/\eta_n ST$ and $N_{\nabla T} = \rho \nabla P_T L_c^3/\eta_n^2 = \rho^2 S \nabla T L_c^3/\eta_n^2$. Although the characteristics of the He II laminar flow are rarely evaluated in practical applications, its analysis helps to establish the dimensionless numbers which will be used as a basic framework for the analyses in the following sections.

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Nomenclature

A_{GM}	Gorter–Mellink constant (turbulent reciprocal viscosity, m s/kg	q	heat flux density, W/cm^2
D	diameter, m	q_L	limiting heat flux density, W/cm^2
$f(T_\lambda)$	a function related to thermodynamic properties at the lambda point	S	entropy, J/kg K
$g(T)$	a function related to thermodynamic properties of He II	S_λ	entropy at the lambda point, J/kg K
K_{GM}	Universal Gorter–Mellink constant	T	temperature, K
L	length, m	T_λ	temperature at the lambda point, K
L_c	characteristic length, m	$y(T_\lambda)$	a function related to thermodynamic properties at the lambda point
N_q	dimensionless number related to the heat flux density	$z(t_b)$	a function related to thermodynamic properties of He II
$N_{grad\,T}$	dimensionless number associated with the temperature gradient	ρ	density, kg/m^3
P	pressure, bar	ρ_n	normal fluid density, kg/m^3
		ρ_s	superfluid density, kg/m^3
		η_n	normal fluid shear viscosity, kg/m s
		η_λ	shear viscosity at the lambda point, kg/m s

3. Transport at small ΔT-values ($\Delta T \ll T$)

The counter flow of GM-transport in an insulated duct is usually written as

$$q = \rho_s S T \left\{ \frac{S|\nabla T|}{A_{GM}\eta_n} \right\}^{1/3} \tag{2}$$

(S = entropy per unit mass, η_n is the shear viscosity of the normal fluid). There are two asymptotes for A_{GM}, one at low temperatures, the other one at high temperatures. Soloski and Frederking [3] demonstrated that at low temperatures, the macroscopic continuum approaches led to $(\eta_n A_{GM})^{-1} = \text{constant} = K_{GM}^3$, where K_{GM} is of the order $10(\pi)^{1/3}$. At higher temperatures, near the lambda temperature (T_λ) A_{GM} approaches the function (ρ/ρ_s). For both constraints to be satisfied, the GM-transport property becomes

$$A_{GM} = K_{GM}^{-3}(\rho/\rho_s)/\eta_n \tag{3}$$

Insertion of Eq. (3) into (2) gives

$$q = K_{GM}\rho_s S T \left\{ \left(\frac{\rho_s}{\rho_n}\right)\left(\frac{\eta_n}{\rho}\right) S \mid \nabla T \mid \right\}^{1/3} \tag{4}$$

For the ease of estimating the heat flux density and the temperature gradient for a given GM duct design, one can use the reduced thermo physical properties for calculation. The reduced properties are evaluated with respect to the properties at the lambda point. The relationship between the reduced shear viscosity and the reduced temperature for various pressures is shown in Fig. 1 [4–7]. It is noted that the appearance and the shape of the reduce viscosity is substantially similar at different pressures. Eq. (4) can be further simplified so that an effective thermal conductance can be derived against reduced thermal physical properties.

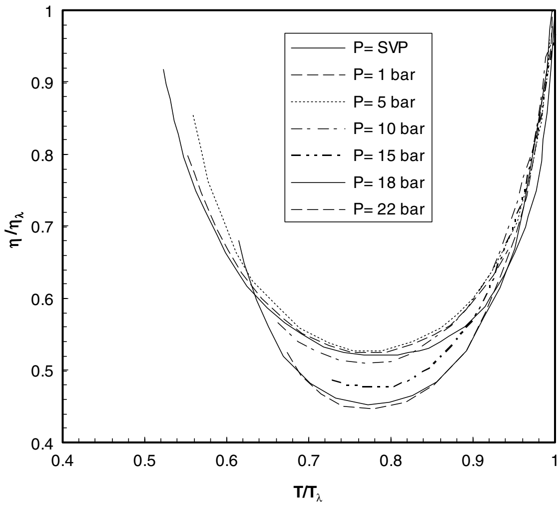

Fig. 1. The reduced viscosity versus the reduced temperature of He II at different pressures.

$$\frac{q^3}{\nabla T} = K_{GM}^3 \rho_\lambda^2 S_\lambda^4 T_\lambda^3 \eta_\lambda \times \left\{ \left(1 - \frac{\rho_n}{\rho}\right)^4 \left(\frac{\rho_n}{\rho}\right)^{-1} \left(\frac{\rho}{\rho_\lambda}\right)^2 \left(\frac{S}{S_\lambda}\right)^4 \left(\frac{T}{T_\lambda}\right)^3 \left(\frac{\eta}{\eta_\lambda}\right) \right\} \tag{5}$$

Apparently, it is easier to evaluate the lambda point properties and the dimensionless values separately. The following two functions are defined for the ease of calculation.

$$f(T_\lambda) = \left(\rho_\lambda^2 S_\lambda^4 T_\lambda^3 \eta_\lambda\right) \tag{6}$$