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Vertical stability of bubble chain: Multiscale approach

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Abstract

Linear stability is investigated of a uniform chain of equal spherical gas bubbles rising vertically in unbounded stagnant liquid at Reynolds number $Re = 50$–200 and bubble spacing $s > 2.6$ bubble radii. The equilibrium bubble positions are questioned for their stability with respect to small displacements in the vertical direction, parallel to the chain motion. The transverse displacements are not considered, and the chain is assumed to be laterally stable. The bubbles are subjected to three kinds of forces: buoyant, viscous, inviscid. The viscous and inviscid forces have both pairwise (local) and distant (nonlocal) components. The pairwise forces are expressed by the leading-order formulas known from the literature. The distant forces are expressed as a linear superposition of the pairwise forces taken over several farther neighbours. The stability problem is addressed on three different length scales corresponding to: discrete chain (microscale), continuous chain (mesoscale), bubbly chain flow (macroscale). The relevant governing equations are derived for each scale. The microscale equations are a set of ODE's, the Newton force laws for the individual discrete bubbles. The mesoscale equation is a PDE for bubbles continuously distributed along a line, obtained by taking the continuum limit of the microscale equations. The macroscale equations are two PDEs, the mass and momentum conservation equations, for an ensemble of noninteracting mesoscale chains rising in parallel. This transparent two-step process (micro → meso → macro) is an alternative to the usual one-step averaging, in obtaining the macroscale equations from microscale information. Here, the scale-up methodology is demonstrated for 1D motion of bubbles, but it can be used for behaviour of 2D and 3D lattices of bubbles, drops, and solids.

It is found that the uniform equilibrium spacing results from a balance between the attractive and repulsive forces. On all three length scales, the equilibrium is stabilized by the viscous drag force, and destabilized by the viscous shielding force (*shielding instability*). The inviscid forces are stability neutral and

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generate conservative oscillations and concentration waves. The stability region in the parameter plane $s - Re$ is determined for each length scale. The stable region is relatively small on the microscale, larger on the mesoscale, and shrinks to zero on the macroscale where the bubbly chain flow is inherently unstable.

The *shielding instability* is expected to occur typically in intermediate *Re* flows where the vertical bubble interactions dominate over the horizontal interactions. This new kind of instability is studied here in a great detail, likely for the first time. Its relation to the elasticity properties of bubbly suspension on different length scales is discussed too. The shielding force takes the form of a negative bulk modulus of elasticity of the bubbly mixture.

Keywords: Bubble chain; Stability; Distant coupling; Multiscale; Shielding instability; Bulk elasticity

1. Introduction

One-dimensional arrays of interacting particles are of high interest, both on their own, and as a prelude to two and three spatial dimensions (Bernasconi and Schneider, 1981). The one-dimensionality usually makes the problem tractable or at least near-tractable (Mattis, 1993). This holds equally well for dispersed particles subjected to hydrodynamic forces. In-line interactions commonly occur in various flow situations, e.g. aerosols and atmospheric problems (Pruppacher and Klett, 1998), sprays and combustion (Sirignano, 1999), granular flows (Hinch and Saint-Jean, 1999), sedimentation (Happel and Brenner, 1965; Dixon et al., 1976), fluidized beds (Werther, 1977; Foscolo and Gibilaro, 1984; Broadhurst, 1986), and also in gas–liquid systems (e.g. Harper, 2001; Liger-Belair and Jeandet, 2002), which is the topic of the present study. The uniform bubble chain possesses the translational symmetry, where all particles are subjected to the same force law. Ideally, the chain is infinite. However, finiteness is needed both in calculations and measurements. Therefore, the fixed-end boundary condition is employed in modelling, and the continuous generation of bubbles in experiments. Both correspond to observing only a finite segment within a virtually infinite chain.

Continuously generated uniformly spaced bubble chains passing through finite layers of quiescent liquids do exist and have been observed in many experiments, in ranges of the Reynolds and other relevant numbers. This means that they are both vertically and laterally linearly stable; otherwise we could not produce them. The bubble formation process can be quite complicated in real systems, and sophisticated ways of bubble generation have to be used to control it. Usually, the bubble size and spontaneous formation frequency have been measured (Coppock and Meiklejohn, 1951) and coalescence phenomena studied (Nevers and Wu, 1971). Recently, acoustic emissions generated by a bubble chain has been investigated too (Manasseh et al., 2004). The chain speed is higher than the single bubble speed due to the collective drag reduction (e.g. Miyahara et al., 1984; Zhang and Fan, 2003). This so-called *shielding effect* is particularly strong at low bubble spacing, s less than 10, say. The shielding is typical for the strictly in-line arrangement where a bubble travels in the wake of the preceding bubble (local interaction), or, more generally, in the liquid disturbed by all preceding bubbles (distant interactions). The shielding effect is in a severe contrast with the *hindrance* effect that results in higher collective drag of bubbles rising in general positions, which is the usual case of real bubble suspensions (e.g. Richardson and Zaki, 1954), see Fig. 2.