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Dynamic models of residential segregation: An analytical solution

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ABSTRACT

We propose an analytical solution to a Schelling segregation model for a relatively broad range of utility functions. Using evolutionary game theory, we provide existence conditions for a potential function, which characterizes the global configuration of the city and is maximized in the stationary state. We use this potential function to analyze the outcome of the model for three utility functions corresponding to different degrees of preference for mixed neighborhoods: (i) we show that linear utility functions is the only case where the potential function is proportional to collective utility, the latter being therefore maximized in stationary configurations; (ii) Schelling's original utility function is shown to drive segregation at the expense of collective utility; (iii) if agents have a strict preference for mixed neighborhoods but also prefer to be in the majority versus the minority, the model converges to perfectly segregated configurations, which clearly diverge from the social optimum. Departing from the existing literature, these conclusions are based on analytical results which open the way to analysis of many preference structures. Since our model is based on bounded rather than continuous neighborhoods as in Schelling's original model, we discuss the differences generated by the bounded- and continuous-neighborhood definitions and show that, in the case of the continuous neighborhood, a potential function exists if and only if the utility functions are linear. A side result is that our analysis builds a bridge between Schelling's model and the Duncan and Duncan segregation index.

1. Introduction

Ethnic and immigrant residential segregation is a striking feature of most Western cities. Extensive views of segregation patterns in the U.S. are provided in Cutler et al. (2008), Iceland and Scopilliti (2008) and Reardon et al. (2008). Cutler et al. (2008) examine a range of potential determinants of immigrant segregation, including their cultural traits and nationalist sentiments among U.S. natives. Card et al. (2008) findings for racial segregation in the period 1970–2000 show evidence of tipping-like behaviors: an increase in the minority share in a neighborhood above a certain threshold leads to a further decrease in the native (in their case white) population. This analysis is one of the first to provide clear empirical evidence of non-linear dynamic aggregate behaviors predicted by social interaction models. According to these results, the utility of the white population in a mixed neighborhood seems to exhibit a sharp decrease beyond a certain minority share. The authors also provide evidence of a direct link between white's attitudes toward minority members and aggregate configurations, as measured by the location of the tipping point. However, the theoretical relationship between individual preferences and aggregate configurations has not been fully explored.

Schelling provided an early contribution, proposing a model to formalize the aggregate consequences of individual preferences related to the social environment (Schelling, 1969, 1971, 1978). The two basic components of Schelling's “Dynamic models of segregation” (1971) are an individual utility function that entirely determines the level of satisfaction enjoyed by an agent in a location, and a dynamical rule that drives agents' location changes and, therefore, the way that the configuration of the city evolves. Using an inductive approach, Schelling shows that if the preferences considered are such that an environment of more than 50% of own-group agents is highly preferred to a less than 50% of own-group environment, then the equilibrium configuration exhibits high levels of segregation, although there is no preference for segregation *per se*. Schelling's (1971) paper is a seminal work in relation to his seemingly paradoxical finding that mild individual preferences for own-group neighbors lead to complete segregation at the global level. However, on reflection, we realize that, given the highly asymmetrical utility function used in this model, it could hardly lead to an integrated environment. Nevertheless, later

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research shows that even a peaked utility function, that is, a function achieving its maximum for a perfectly mixed environment, can lead to a fully segregated equilibrium as soon as this function is asymmetric (Zhang, 2004b; Pancs and Vriend, 2007; Barr and Tassier, 2008).

The realism of Schelling's model can be criticized on the grounds that it ignores institutional causes of segregation, income effects, and the social structure of the city. Nevertheless, the model is popular for modeling social systems to show the unintended macro-level consequences of individual behavior, and Schelling's 1971 paper has become the most widely cited of his contributions (more than 460 citations to it on June 10, 2010). For many years there were relatively few citations to it, but since 2003 there have been around 40 citations per year, demonstrating the renewed interest in Schelling's model. It is interesting also that it is cited in widely different fields: economics and sociology are responsible for the highest number (40%) followed by computer science, mathematics and physics which together account for 24% of the citations to this paper. This substantial scientific activity has led to new insights: interpretations of the emergence of segregation patterns as the result of a coordination problem (Zhang, 2004a,b); a physical analog of Schelling's model (Vinkovic and Kirman, 2006); evidence of the robustness of Schelling's results with respect to different definitions of individual utilities and/or environments (Pancs and Vriend, 2007; Fagiolo et al., 2007); the impact of heterogeneous agents and public policies (O'Sullivan, 2009); the exploration of tipping behaviors (Zhang, 2011).[1]

Attempts to solve Schelling's model analytically include Dokumaci and Sandholm (2007); Mobius and Rosenblat (2000); Pollicott and Weiss (2001); Pancs and Vriend (2007); Zhang (2004a,b, 2011). Zhang (2004a,b, 2011)'s contributions are the most successful of these so far. Zhang proposes variations to Schelling's model which he analyzes formally using the concept of potential function developed in evolutionary game theory. Zhang (2004a) considers a model with vacant cells and linear utility functions. Zhang (2004b, 2011) use an asymmetric peaked utility function in a model with no vacant cells. The latter raises the issue of individual rationality, since it is assumed that, for individual moves to occur, two agents must coordinate and agree to exchange locations. This analysis, then, departs from Schelling's original framework. Also, the three contributions referred to above cover only two specific utility functions.

In the present paper, we build on Zhang (2004a,b, 2011). We transpose Schelling's model to an evolutionary game theory context with the aim of characterizing the equilibrium segregation level by means of a potential function. In contrast to Zhang (2004b, 2011), we consider bounded neighborhoods, i.e. blocks where all agents share the same neighbors. In this context, we show that a potential function of the model exists if and only if the utility functions are such that the externalities generated by one type of agent are symmetric to those generated by the other type of agent or in the limit case where there are no vacant cells. Under this condition, we can identify a general form of the potential function and can predict the global pattern emerging from different utility functions, which, to the best of our knowledge, has not previously been done. The main property of this potential function is that it reflects both the macro and micro scale. On the one hand, this aggregate function depends only on the number of agents of each type in each block. On the other hand, it keeps tracks of the individual level since it corresponds to the sum of the individual utility changes generated by individual moves.

We use this potential function to characterize the segregation level of the stationary configurations of the model for different utility functions, representing different degrees of preference for mixed environments. We examine successively: (i) linear utility functions, with a continuous preference for segregated environments; (ii) Schelling's original utility function in which there is a mild preference for a mixed environment; and (iii) asymmetric peaked utility functions, according to which agents clearly exhibit a preference for a mixed environment. We show that there is no divergence between individual moves and social welfare with increasing linear utility functions, although segregation prevails in stationary configurations. We show also that even with the strongest preference for a mixed environment – in the asymmetric peaked utility function case – the model yields segregated stationary configurations. In this case, too, the divergence between the stationary segregation level and the optimal segregation level is highest. These results complement those obtained by Zhang (2004b, 2011) and Pancs and Vriend (2007).

In summary, our work provides a relatively general solution to Schelling's model with bounded neighborhoods, that encompasses previous work on this model and opens the way to analysis of many structures of preferences, for instance, those based on empirical findings concerning racial preferences. We show a few simulation results by way of illustration.

This paper is organized as follows. The model features are presented in Section 2. Section 3 defines the potential function concept and states our main result. The potential function is applied in Section 4 to study the stationary configurations obtained for three different utility functions. In one of these cases, we are able to build a connection with a commonly used segregation measure. In Section 5, we demonstrate the supplementary result that a potential function exists with continuous neighborhoods if and only if the utility functions are linear, and we discuss the differences between the bounded and continuous neighborhood cases. We also consider agents' preferences for local amenities and analyze a case where a tax on segregative behaviors is introduced.

2. A general dynamic model of segregation

2.1. The city and the agents

Our artificial city is a two-dimensional $N \times N$ square lattice with periodic boundary conditions, i.e. a torus containing N^2 cells. Each cell corresponds to a dwelling (residential unit) and all are of equal quality. We suppose that a certain characteristic divides the population of this city into two groups of households, which we will refer to as red and green agents. Each location may be occupied by a red agent, a green agent, or be vacant. We denote the number of vacant cells as N_V, and the number of respectively red and green agents as N_R and N_G. Thus the parameter N controls the size of the city, the parameter $v = N_V/N^2$ its vacancy rate, and the fraction $n_R = N_R/(N_R + N_G)$ its composition.

We define a *state* x of the city as a N^2-vector, each element of this vector labeling a cell of the $N \times N$ lattice. Each state x thus represents a specific configuration of the city. X is the set of all possible configurations, the demographic parameters (N, v, n_R) being fixed.

2.2. Neighborhoods

Since Schelling (1969)'s work, two ways of conceiving the neighborhood of an agent have been developed and used in analytical and simulation models.

Bounded neighborhood models (Fig. 1a) describe cities divided into geographical units within which all agents are connected. The neighborhood of an agent therefore is composed entirely and exclusively of the locations in the same geographical unit as his own. In the following, when we refer to a bounded neighborhood model, we assume implicitly that the city is divided into a set $\mathcal{Q}$ of blocks, each of which contains $H+1$ locations, where H is a fixed integer that corresponds to the number of locations in an agent's neighborhood (hence, the relation $|\mathcal{Q}|(H+1) = N^2$ must hold). Obviously, the description of the city as a lattice with periodic boundary conditions is unnecessary in this case. Note that since some locations remain empty, the size H of the neighborhood of an agent can also be interpreted as the maximum number of neighbors an agent can have. For a given configuration $x \in X$ of the city, we use $R_q(x)$ and $G_q(x)$

[1] See Clark and Fossett (2008) for a literature review.