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Convective behaviour of a uniformly Joule-heated liquid pool in a rectangular cavity

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Abstract

A two-dimensional mathematical model has been developed to study the interaction between gravitational body force and self-induced electromagnetic body force in a Joule-heated liquid pool in a rectangular cavity, with an aspect ratio of 2. The Joule heating of the liquid pool in the cavity is accomplished by passing a large alternating current employing a pair of plate electrodes immersed in the liquid. The electrode surfaces are assumed to be isopotential and rest of the boundaries is treated as electrically insulated. The upper boundary of the liquid pool is an isothermal boundary while the rest of the boundaries are assumed to be thermally insulated. The present study investigates the convective behaviour of different liquids under Joule heating. Numerical simulations have been carried out employing Boussinesq fluids for Rayleigh numbers up to 2.5×10^5 and Hartmann numbers up to 4×10^7. This study shows that the heat transfer in the uniformly Joule-heated liquid is governed by the gravitational body force when $Ha^2 Pr/\sqrt{Ra} < 120$ and by the self-induced electromagnetic body force when $Ha^2 Pr/Ra > 100$. It also indicates that the thermal field is strongly dependent on Pr in electromagnetically driven flows while Pr has negligible effect on temperature field in thermally driven flows for $10^3 \geqslant Pr \geqslant 10$. Heat transfer correlations for thermally driven flows, electromagnetically driven flows and combined flows are also presented.

Keywords: Joule heating; Internal heat generation; Natural convection; Magnetohydrodynamic flow; Heat transfer

1. Introduction

The principle of Joule heating is extensively made use of in the electrothermal industries to give rise to elevated process temperatures. For example, Joule heating is used for melting glasses in electric glass melters and for heating molten slag in electro-slag remelting process applied in the steel industry. The above electrothermal processes employ either direct current or low frequency, alternating currents for heating the process medium, which is in liquid state placed between a pair of electrodes.

Physical phenomena taking place inside Joule-heated liquid pools are quite complex and interrelated. The heat transfer in the liquid pool is governed by the natural convection caused by the body forces generated on account of Joule heating. Two types of body forces prevail in an electrically conducting liquid under Joule heating. These are the *gravitational body force* due to the non-uniform temperature field and the *electromagnetic body force* due to the interaction between self-induced magnetic field and moving charge carriers in the liquid. The convective behaviour of the liquid pool under Joule heating depends strongly on the interaction between the two body forces.

Mathematical models have been developed by various researchers to study the convective behaviour of liquids under Joule heating [1–5]. Models developed for predicting the flow field and the temperature distribution inside the

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Nomenclature

$\mathbf{A}$	magnetic vector potential	$V \cdot s \cdot m^{-1}$
$\mathbf{B}$	magnetic flux density vector	$Wb \cdot m^{-2}$
$\mathbf{D}$	electric flux density	$C \cdot m^{-2}$
$\mathbf{E}$	electric field intensity	$V \cdot m^{-1}$
$\mathbf{g}$	gravitational acceleration vector	$m \cdot s^{-2}$
g	acceleration due to gravity 9.81	$m \cdot s^{-2}$
h	heat transfer coefficient	$W \cdot m^{-2} \cdot K^{-1}$
$\mathbf{H}$	magnetic field strength	$A \cdot m^{-1}$
$\mathbf{J}$	electric current density	$A \cdot m^{-2}$
k	thermal conductivity	$W \cdot m^{-1} \cdot K^{-1}$
L	vertical dimension of cavity	m
p	excess pressure over static pressure	$N \cdot m^{-2}$
$\mathbf{r}$	position vector	m
t	time	s
T	temperature	K
u	velocity along x direction	$m \cdot s^{-1}$
$\mathbf{V}$	velocity vector	$m \cdot s^{-1}$
w	velocity along the z direction	$m \cdot s^{-1}$
W	horizontal dimension of cavity	m
x, y, z	Cartesian coordinates	m

Greek symbols

α	thermal diffusivity	$m^2 \cdot s^{-1}$
β	coefficient of volumetric expansion	K^{-1}
φ	electric scalar potential	V
μ	magnetic permeability	$H \cdot m^{-1}$
ν	kinematic viscosity	$m^2 \cdot s^{-1}$
ρ	density	$kg \cdot m^{-3}$
σ	electrical conductivity	$mho \cdot m^{-1}$
ω	angular frequency of current	$rad \cdot s^{-1}$
∇	gradient operator	m^{-1}
∇^2	Laplacian operator	m^{-2}
Δ	difference of a quantity	

Superscript

$'$	dummy variable

Subscript

0	reference value
avg	average value
max	maximum value
x, y, z	component of a vector quantity

Dimensionless quantities

$\widetilde{\mathbf{A}}$	magnetic vector potential, $\mathbf{A}W/\sigma\mu\varphi_0 L^2$
$\widetilde{\mathbf{B}}$	magnetic flux density, $\mathbf{B}W/\sigma\mu\varphi_0 L$
$\widetilde{\mathbf{E}}$	electric field intensity, $\mathbf{E}W/\varphi_0$
$\widetilde{\mathbf{J}}$	electric current density, $\mathbf{J}W/\sigma\varphi_0$
$\tilde{p}$	pressure, $pL^2/\rho\nu^2$
$\widetilde{T}$	temperature, $2(T-T_0)kW^2/\sigma\varphi_0^2 L^2$
$\tilde{u}, \tilde{w}$	x- and z-components of $\widetilde{\mathbf{V}}$
$\widetilde{\mathbf{V}}$	velocity vector, $\mathbf{V}L/\nu$
$\tilde{x}$	Cartesian coordinate in x direction, x/L
$\tilde{z}$	Cartesian coordinate in z direction, z/L
$\tilde{\varphi}$	electric scalar potential, φ/φ_0
$\widetilde{\nabla}$	gradient operator, ∇L
$\widetilde{\nabla}^2$	Laplacian operator, $\nabla^2 L^2$

Dimensionless numbers

a	aspect ratio, W/L
Ha	Hartmann number, $J_0 L^2\sqrt{\mu/\rho_0\nu^2} = L\sigma\varphi_0\sqrt{\mu/\rho_0\nu^2 a^2}$
Nu_{avg}	average Nusselt number, $2/\frac{1}{a}\int_0^a \widetilde{T}\,\mathrm{d}\tilde{x}$
N_f	square of the ratio of L to the electromagnetic wavelength, $\omega\mu\sigma L^2$
N_{Rem}	magnetic Reynolds number, $\sigma\mu L V_0 = \sigma\mu\nu$
Pr	Prandtl number, ν/α
Ra	Rayleigh number, $L^3 g\beta\Delta T_0/\nu\alpha = L^3 g\beta\sigma\varphi_0^2/2k\alpha\nu a^2$

electro-slag remelting furnace assume that the gravitational body force can be neglected when compared with the electromagnetic body force [1,2]. On the other hand, the electromagnetic body force is ignored in the models for the electric glass melters [3–5]. Thus, it is evident from the literature that physical properties of electrically conducting liquids strongly govern their convective behaviour under Joule-heating. The present study investigates the convective behaviour of different electrically conducting liquids ranging from molten metal to molten glass. Main aim of this study is to identify the conditions under which each one of these two body forces prevailing in a Joule-heated liquid pool dominates the other. It also characterises convective behaviour of thermally driven flows, electromagnetically driven flows and combined flows in a Joule-heated liquid pool. Heat transfer correlations for these three types of flow are also presented.

2. Governing equations and boundary conditions

Mathematical model to describe the state of an electrically conducting liquid pool in a cavity involves model equations for fluid flow, heat transfer and Maxwell's equations describing the electromagnetic field.

2.1. Electromagnetic field

The Maxwell's equations describing the electromagnetic fields prevailing in a Joule-heated liquid pool can be written as

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{2}$$