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Mixed convection flow along a vertical plate subjected to time-periodic surface temperature oscillations

Nawaf H. Saeid [1]

Mechanical Engineering School, University of Science Malaysia, 14300 Nibong Tebal, Pulau Pinang, Malaysia

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Abstract

The effect of the periodic oscillations of the surface temperature with time on the mixed convection from a vertical plate is investigated. The problem has been simplified by employing the laminar boundary layer and Boussinesq approximations. The fully implicit finite-difference scheme is used to solve the dimensionless governing equations. The results for the laminar flow of air having $Pr = 0.72$ and water having $Pr = 7.00$ are presented for an isothermal flat plate and for the periodic oscillation of the temperature on the plate. The results show the steady periodic variation of the Nusselt number and friction coefficient for both aiding and opposing flows with different amplitudes and frequencies of the oscillating surface temperature. For both aiding and opposing flow, both the Nusselt number and the friction coefficient oscillate with the oscillation of the plate temperature and for some cases (at high amplitudes and frequencies) the Nusselt number becomes negative. For constant Prandtl number and Richardson number, the cyclic average values of the Nusselt number are decreasing with increasing either the amplitude or the frequency of the surface temperature oscillations. The cyclic average values of the friction coefficient, for all the cases considered, are found to be constant and approximately equal to the values for non-oscillating surface temperature.

Keywords: Mixed convection; Periodic convection; Boundary layer; Oscillating temperature; Numerical study

1. Introduction

Mixed (combined free and forced) convection from a vertical flat plate is important in nature and in many practical transport process devices, such as furnaces, electronic devices cooled by external forced circulation, solar collectors, chemical processing equipments and others. In these applications and others, the change in the plate temperature causes free and/or forced convection flow could be a sudden change or a periodic one, leading to a variation in the flow. The literature shows that the steady and transient mixed convection theory is well established and has been investigated by various researchers. Representative studies in this area may be found in the books by Gebhart et al. [1], Bejan [2], Pop and Ingham [3] and Cebeci [4].

The theory of mixed convection shows that the ratio of the Grashof number to the square of Reynolds number (Gr/Re^2) which is known as the Richardson number, has a great influence on the flow for a constant plate temperature. The forced convection is dominating for small values of the Richardson number while free convection takes over for large values of Richardson number. Steady mixed convection from a vertical plate has been studied by various authors, including Merkin [5], Wilks [6], Lloyd and Sparrow [7], Gryzagoridis [8] and Jaluria [9]. A combination of series expansion and numerical integration was used by Merkin [5] to solve the mixed convection problem along an isothermal wall for both aiding (the external flow is in the same direction as the buoyancy force) and opposing (the external flow is in the opposite direction to the buoyancy force) flows for a Prandtl number of unity. Merkin [5] has shown that in the case of opposing flow, as buoyancy effects increase,

E-mail address: n_h_saeid@yahoo.com (N.H. Saeid).
[1] Present address: Department of Mechanical Engineering, Curtin University of Technology, CDT 250, 98009 Miri, Sarawak, Malaysia. Tel: 604-5937788 ext: 6363, fax: 604-5941025.

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Nomenclature

$A(\Re)$	amplitude of $\Re$, Eq. (16)	
C_f	friction coefficient, Eq. (13)	
g	magnitude of the gravitational acceleration	$\mathrm{m \cdot s^{-2}}$
Gr	Grashof number based on L	
L	plate height	m
Nu_x	local Nusselt number, Eq. (14)	
Pr	Prandtl number	
Re	Reynolds number based on L	
Ri	Richardson number, $= Gr/Re^2$	
t	time	s
T	temperature	K
u, v	velocity components	$\mathrm{m \cdot s^{-1}}$
U, V	non-dimensional velocity components	
x, y	Cartesian coordinates	m
X, Y	non-dimensional Cartesian coordinates	

Greek symbols

α	thermal diffusivity	$\mathrm{m^2 \cdot s^{-1}}$
β	coefficient of volume expansion	$\mathrm{K^{-1}}$
ε	non-dimensional amplitude	
ν	kinematic viscosity	$\mathrm{m^2 \cdot s^{-1}}$
θ	non-dimensional temperature	
τ	non-dimensional time	
ω	frequency	$\mathrm{s^{-1}}$
Ω	non-dimensional frequency	
$\Re$	either $Nu_x/\sqrt{Re}$ or $C_{fx}\sqrt{Re}$	

Subscripts

w	wall
∞	free stream

the boundary layer separates from the plate at a Richardson number $= 0.192357$. A similar study has been carried out by Wilks [6] and the results were obtained again for $Pr = 1$ but for a constant heat flux boundary condition. Lloyd and Sparrow [7] have obtained a complete solution to the aiding mixed convection from a vertical isothermal flat plate by using the local similarity method for different values of the Prandtl number $Pr = 0.003$, 0.01, 0.03, 0.72, 10 and 100 and for Richardson number in the range 0 to 4. The aiding mixed convection from a vertical plate has been investigated experimentally by Gryzagoridis [8]. Experimental measurements of the Nusselt number of air ($Pr = 0.72$) are presented by Gryzagoridis [8] for a wide range of Richardson number (0 to 500). A numerical study of aiding mixed convection flow over localized, finite-sized isoflux sources located on a vertical adiabatic vertical surface has been carried out by Jaluria [9]. In this study, the heat transfer coefficient for the upper source is found to depend very strongly on the separation distance between the sources, and increasing this distance leads to an increase in the heat transfer coefficient.

The development in the theory and applications of the mixed convection from the vertical plate has led to an increased interest in the transient and unsteady mixed convection flows. Sammakia et al. [10] studied the transient response of the aiding mixed convection from a vertical flat surface with both a uniform surface temperature and uniform heat flux from the surface for $Pr = 0.72$ and $Pr = 7.6$. They used the explicit finite difference method to solve the dimensionless governing equations and presented the results for the velocity and temperature profiles at different time steps until the steady state is reached. In another study by the same group, Sammakia et al. [11] measured and calculated transient mixed convection from a vertical flat surface with uniform heat flux from the surface for $Pr = 0.72$. A similar transient mixed convection study with different mathematical formulation, using the model given by Yan [12], was carried out by Mai et al. [13] for uniform heat flux boundary condition. Mai et al. [13] have employed the implicit finite-difference scheme to solve the governing equations for both transient aiding and opposing flows and present the velocity and temperature profiles for different time steps until the steady state is reached. Zubair and Kadaba [14] have constructed similarity variables of the transient mixed convection problem and obtained new solutions. Merkin and Pop [15] and Steinruck [16] have shown that the similarity solutions of the boundary-layer flow equations describing mixed convection flow along a vertical plate exists if the difference between the temperature of the plate and the ambient temperature is inversely proportional to the distance from the leading edge of the plate. The effect of wall conduction on the characteristics of unsteady mixed convection is important in the engineering applications. These types of unsteady conjugated mixed convection problems were studied numerically by Yan and Lee [17] and Lee and Yan [18] for vertical channel flows.

It is noted that for steady state and transient mixed convection problems, the constant, streamwise surface temperature variation or constant heat flux are usually assumed in the above mentioned studies. However, in industrial applications, quite often the convection heat transfer is a periodic process. Saeid [19] has studied the periodic free convection from a vertical plate. The laminar boundary layer theory is used to study the effect of periodic plate temperature oscillations with different amplitudes and frequencies on the free convection flow with different Prandtl numbers. Recently Saeid and Mohamad [20] have considered the surface temperature oscillation in free convection from a vertical plate immersed in porous media. In the above two studies, [19] and [20], it is found that the free convection heat transfer from the vertical plate decreases with an increase in either the amplitude or frequency of the surface temperature oscillation. In this paper the problem studied by Saeid