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A fuzzy approach to the convective longitudinal fin array design

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Abstract

This work considers an optimum design problem for the longitudinal fin array with constant heat transfer coefficient in a fuzzy environment, where rigid requirements to strictly satisfy the total fin volume and array width and maximize the heat dissipation rate are softened. The proposed method shows that the fuzzy fin array design problem can be converted into a regular min–max type optimization problem by employing the tolerance approach. An entropic regularization technique is then applied to solve the resulting optimization problem. Some computational results are presented to illustrate the theory and solution procedure. A comparison of results from the regular non-fuzzy optimal model and the fuzzy model is also included.

Keywords: Fin array; Fuzzy approach; min–max optimization; Tolerance approach; Entropic regularization technique

1. Introduction

It is well known that fins have been widely used in heat transfer devices to increase the heat dissipation rate. The fin design optimization problem has great interest and much work has been done to improve fin performance in the literature. There are two different approaches for these fin optimization problems. In the first approach, the minimum volume or least fin fabrication material is expanded for the prescribed heat dissipation. The other approach considers finding the maximum heat dissipation for a given fin volume. Most of these works were performed using a single fin or spine. A thorough treatment for the optimum single fin or spine design was given by Kern and Kraus [1]. However, fin arrays are used more often than single fins or spines in practical engineering applications. Dhar and Arora [2] presented methods for carrying out the minimum fin array weight design with triangular cross-section for a flat surface, cylindrical surface, and rectangular cross-section over a flat surface. Bar-Cohen [3] studied the fin thickness for a natural convective fin array with a rectangular profile and concluded that in natural convection arrays, superior thermal performance is generally associated with relatively thick fins. A least material optimization investigation of a vertical plate-fin heat sink in natural convective heat transfer was given by Iyengar and Bar-Cohen [4]. The optimum dimensions of the fin space and fin thickness were studied using the Nusselt number correction by Bar-Cohen and Rohsenow. Recently, Bar-Cohen et al. [5] extended the least-material single rectangular plate-fin analysis to multiple fin arrays, using a composite Nusselt number correlation to find the globally best thermal design for the natural convective heat sinks. In their work, optimum fin array design was combined with the least fin material and optimal spacing.

The development of fuzzy set theory has forged a new way to deal with imprecision and vagueness in information since 1965 [6]. It has applications in many different practical fields, such as electrical, air-conditioning and refrigeration, aerospace, chemical, transportation, power industries. The first attempt to apply the fuzzy set theory to a heat transfer problem was Zhang and Chung [7]. The optimum fin height and fin thickness were investigated for a given heat dissipation rate on a single rectangular longitudinal convective fin. The same analysis was applied to a conical convective spine by Chen and Lin [8]. Latter, Chung and Chen [9,10] applied

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Nomenclature

b	half fin thickness	m
$\widetilde{D}$	fuzzy decision	
d	half space between fins	m
H	height of the fin array	m
h	average heat transfer coefficient	$W \cdot m^{-2} \cdot K^{-1}$
h_f	heat transfer coefficient of fin surface	$W \cdot m^{-2} \cdot K^{-1}$
h_H	heat transfer coefficient of fin base	$W \cdot m^{-2} \cdot K^{-1}$
k	thermal conductivity of the fin	$W \cdot m^{-1} \cdot K^{-1}$
L	length of the fin array	m
N	total number of fins	
Q	heat transfer rate of the fin array	W
T	temperature of fin surface	K
t	tolerance	
V	total volume of the fin array	m^3
W	width of fin array	m

Greek symbols

θ	temperature difference between fin surface and ambient	K
μ	membership function	

Subscripts, superscript

H	fin base
∞	ambient
$*$	prescribed value

the fuzzy optimization technique to the rectangular longitudinal fin and cylindrical spine designs with different models. The first paper that studied the fuzzy fin array optimization was presented by Chung et al. [11]. In their work, a four-fin radiating fin array system was developed and optimized by employing the fuzzy approach, in which the total weight and horizontal dimension of the fin system were minimized. This study considers fuzzy optimal longitudinal fin array design for a given array volume and a prescribed array width in a natural convective environment where the heat transfer coefficient are averaged and assumed constant.

As mentioned above, this work studies fuzzy optimum natural convection fin array design with constant heat transfer coefficient. We aim to maximize the heat transfer rate for the given fin volume and array width in accordance with a prescribed tolerance. In our problem, the total volume V and the width W of the fin array are considered to be "approximately" equal to prescribed values V^* and W^*. To investigate the solution method for solving the fuzzy convective fin array design problem, mathematical model thermal analysis is presented in Section 2. Applying the fuzzy set theory, in Section 3 we show that the fuzzy convective fin array design problem can be converted into a regular min–max nonlinear programming problem. An entropic regularization technique [12,13] is then applied to solve the resulting optimization problem. Numerical results and discussion are provided in Section 4 to confirm the efficiency of the proposed method. Section 5 concludes this paper by making some remarks.

2. The thermal analysis

Consider the longitudinal fin array with rectangular cross-section as shown in Fig. 1. The heat dissipation of the fin array is comprised of two components. One is the heat transfer from the surface between the fins, and the other is heat

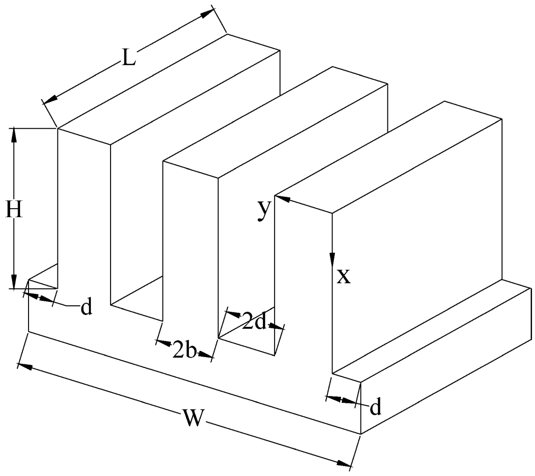

Fig. 1. Schematic of an array with longitudinal rectangular fins.

dissipation from the surface of each fin in the array. In this case, the heat dissipation rate can be described as follows [1]

$$Q = N\left[2dh_H(T_H - T_\infty) + \int_0^H 2h_f(T - T_\infty)\,\mathrm{d}x\right] \tag{1}$$

where N is the total number of fins, d is the half space between the fins, H is the fin height, h_H and h_f are the convection coefficient at the fin base and surface, respectively, T_H is the temperature at the fin base, and T_∞ is the surrounding temperature. In this study, the functional relations of the width, W, and the total volume, V, of the fin array are given as follows for unit fin length $L = 1$.

$$W = N(2b + 2d) = 2N(b + d) \tag{2}$$

$$V = N(2bHL) = \frac{bHW}{b + d} \tag{3}$$

where b is the half fin thickness.

Assuming that the heat conduction along a single rectangular fin is one-dimensional, i.e., along the x direction. The heat entering the fin by conduction is equal to the heat dis-