Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

International Journal of Thermal Sciences 44 (2005) 53–63

International Journal of Thermal Sciences

www.elsevier.com/locate/ijts

Tree networks for minimal pumping power

Louis Gosselin [a,b,*], Adrian Bejan [a]

[a] *Department of Mechanical Engineering and Materials Science, Box 90300, Duke University, Durham, NC 27708-0300, USA*
[b] *Département de génie mécanique, Université Laval, Québec, Québec G1K 7P4, Canada*

Received 8 February 2004; received in revised form 14 April 2004; accepted 30 June 2004

Available online 12 September 2004

Abstract

In this paper the optimization of fluid networks is based on the minimization of pumping power requirement. The total pipe network volume is constrained. It is shown that only in special cases the minimization of pumping power leads to the same architecture as the minimization of pressure drop or flow resistance. Fundamentals of fluid network optimization are developed for both spanning networks and networks where new non-consumer points are added (Gilbert–Steiner points). It is shown that networks with minimum pumping power must not contain loops. The influence of gravity on the optimization of flow configuration is also addressed. The principles developed in the paper are illustrated with an example representing a set of ten vertices to be connected with pipes. The paper provides designers with more effective basic tools for the conceptual design of fluid networks.

Keywords: Tree networks; Dendritic; Constructal theory; Pumping power; Geometry optimization

1. Introduction

Constructal theory began with the problem of distributing high conductivity material and fast routes (streets) for maximizing the access for heat flow and traffic [1–3]. Dendritic fluid flow structures came next, and were generated based on a deterministic principle—the minimization of global flow resistance, subject to global constraints. The tree-shaped flow architecture emerged as a result, not as an assumption. For this reason the constructal method is unlike the fractal approach, in which the algorithms that generate the geometry are postulated.

The constructal method has been applied to the design of tree networks that transport things other than heat and fluid, for example, electricity, people and goods. This work is reviewed in Ref. [1]. Constructal fluid trees are particularly important because of numerous natural flows that display dendritic architectures, e.g., respiratory air ways, vascularized tissues, lightning, river basins and deltas, and rapid solidification (snowflakes).

In this paper we take a fresh look at the generation of tree architectures for fluid flow, and instead of minimizing the global flow resistance we focus on the minimization of pumping power. It is pumping power, or the minimization of exergy destruction (fuel, food) that governs all the complex flow structures that strive for higher efficiency and persistence (survival) in engineering and nature.

2. The choice of pumping power as a cost function

The very idea of system optimization (in engineering as well as in Nature) implies that the system in question is not purposeless: the system has an objective, a duty to fulfill. This task is accomplished at a certain cost, and under global constraints. Identifying these constraints and objectives is the first conceptual step in the process of designing a system. It is a crucial step that calls for adequate modeling. A flawed cost function may lead to “an optimal” design (optimal in the sense that it minimizes the flawed cost function), but there is

* Corresponding author. Tel.: +1-418-656-7829, fax: +1-418-656-7415.
E-mail address: louis.gosselin@gmc.ulaval.ca (L. Gosselin).

1290-0729/$ – see front matter
doi:10.1016/j.ijthermalsci.2004.06.004

Nomenclature

a	cross-sectional area	m^2
f	friction factor	
g	gravity acceleration	$m \cdot s^{-2}$
I	ratio of pumping power requirement with and without loops	
L	length	m
M	mass conservation equation	
$\dot{m}$	mass flow rate	$kg \cdot s^{-1}$
N	number of vertices	
P	pressure	$N \cdot m^{-2}$
$\dot{s}$	vertex fluid consumption	$kg \cdot s^{-1}$
V	total pipe volume	m^3
$\dot{W}$	pumping power requirement	W
z	altitude	m

Greek symbols

α	parameter characterizing cost per unit length of a pipe	
β	Lagrange multiplier	
ε	exponent to obtain pumping power	
λ	Lagrange multiplier	
ν	kinematic viscosity	$m^2 \cdot s^{-1}$
ρ	density	$kg \cdot m^{-3}$
Γ	path from larger to lower pressures	
$\Omega(\dot{m})$	number of pipes with mass flow rate $\dot{m}$	

Superscript and subscripts

$\sim$	dimensionless parameters
e, k	pipe
i, j	vertex indices

no guarantee that this design provides a satisfactory performance in view of the "real" cost function.

In recent research works on fluid networks (e.g., Refs. [4–7]), pressure drop has been used extensively as a measure of the network operation cost. Optimal networks were generated by minimizing the total pressure drop between the points of highest and lowest pressure. In this section, we argue that this cost function is not always the best choice. In place of pressure drop, pumping power, or destroyed exergy—i.e., in the end what really costs to operate the network—can be used as a more realistic cost function. We show that only in special cases the two cost functions are equivalent leading to the same optimal performance and geometric configuration.

To begin with, consider the simplest situation: a fully developed laminar flow in a pipe with circular cross-section. Given the mass flow rate in the pipe, $\dot{m}$, the pressure drop in the pipe and required pumping power are given by

$$\Delta P = \frac{8\pi \nu \dot{m} L}{a^2} \tag{1}$$

$$\dot{W} = \frac{8\pi \nu \dot{m}^2 L}{\rho a^2} \tag{2}$$

where a and L are respectively the cross-sectional area and length of the pipe. The effect of the pipe geometry (a, L) on the pressure drop and pumping power is the same in this particular case: according to Eqs. (1) and (2), ΔP and $\dot{W}$ are both proportional to (L/a^2).

The situation is different when several pipes are connected together to form a network. In that case, the total pressure drop and pumping power can be written as

$$\Delta P = 8\pi \nu \sum_{e \in \Gamma} \frac{\dot{m}_e L_e}{a_e^2} \tag{3}$$

$$\dot{W} = \frac{8\pi \nu}{\rho} \sum_{e} \frac{\dot{m}_e^2 L_e}{a_e^2} \tag{4}$$

The summation in Eq. (3) is over the pipes e of a path Γ (i.e., over only some of the pipes of the network) from the point of largest pressure to the point of smallest pressure. The summation in Eq. (4) is over all the pipes of the network. It can be shown that the summations in Eqs. (3) and (4) are equivalent if: (i) all the pipes of the network (with a given mass flow rate) have the same length and cross-sectional area; (ii) the quantity $\dot{m} \cdot \Omega(\dot{m})$ is a constant independent of $\dot{m}$, where $\Omega(\dot{m})$ is the number of pipes with a mass flow rate $\dot{m}$.

When conditions (i) and (ii) described above are not respected, pumping power and pressure drop minimization will lead to different network configurations and levels of performance. Therefore, in general, one cannot presume that a minimum pressure drop network will look or perform as a minimum pumping power network.

To illustrate the difference between the two approaches, consider a simple network with one source and two fluid users i and j positioned on a single line, as shown in Fig. 1. The first pipe connects the source with the vertex i, while the second pipe connects i with j. We assume that the points i and j consume the same amount of fluid. Therefore, the mass flow rate in the first pipe is twice as large as in the second pipe. The cross-sectional areas of the two pipes are the parameters to optimize. However, in view of the total pipe volume constraint (see Eq. (7)), there is only one independent variable. We chose the cross-sectional area of the first pipe ($\tilde{a}_1$) as this degree of freedom. According to Eqs. (1) and (2), the total pressure drop and pumping power requirement are:

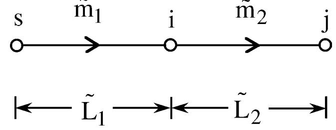

Fig. 1. The discrepancy between pumping power minimization and pressure drop minimization in a simple network with three points.