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Continuum modelling of viscoelastic behavior under squeezing solicitations of electrorheological fluid

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Abstract

In this paper, we propose a continuous model for the electromechanical behavior of the electrorheological fluids in solid state subjected to oscillatory squeeze loads below the chains rupture yield. Our starting point was the experiment led by Vieira et al. [S.L. Vieira, M. Nakano, R. Oke, T. Nagata, Mechanical properties of an ER fluid in tensile, compression and oscillatory squeeze tests, Int. J. Mod. Phys. B 15 (2001) 714–722] and the description of continua whom electromechanical properties were developed, among others, by Eringen and Maugin [G.A. Maugin, A.C. Eringen, On the equations of the electrodynamics of deformable bodies of finite extent, J. Méc. 16 (1977) 101–147; A.C. Eringen, G.A. Maugin, Electrodynamics of Continua, I: Foundations and Solid Media, Springer-Verlag, New York, 1990]. This work has been limited to small mechanical and electric perturbations of a dielectric solid free of mobile electric charges and magnetic phenomena. Similarly to works done for small thermal and mechanical perturbations in thermoelasticity, we suppose that the mechanical behavior is not coupled with the electric one. The dissipative phenomena which are experimentally observed, are modeled by an internal variable which represents the chains microscopic deformation. We present a finite element resolution to our problem, based on a variational formulation using displacement, electric potential and the internal variable. We identify our model from the inverse analysis of Vieira's tests simulation. Next, we simulate the influence of a small perturbation of the electric field on the mechanical response of the material.

Keywords: Electrorheology; Viscoelasticity; Internal variable; Squeezing test simulation; Dielectric; Smart materials

1. Introduction

W.M. Winslow, in 1947, was the first to highlight what we nowadays call the electrorheological effect. Materials with such a property are suspensions whom behavior is greatly modified in the presence of an electric field. That change of behavior is linked to the formation of fibrous structures. Several experimental studies are concerned with this transformation. We can quote the works of Klingenberg and Zukoski [4] and more recently Wen et al. [6]. The suspension is initially placed between electrodes. An electric potential difference is applied, then the particles, whom dielectric constant (electric permittivity) is greater than suspending fluid's one, are polarized and mutually attracted. Dispersed at first, they migrate and form short chains in the electric field direction and coalesce each together constituting fibers having a more or less regular shape. For sufficient electric field and concentration, those fibers can reach the electrodes. Then the materials change from a fluid state to a solid state [7]. This transition is obviously faster as the required displacement of particles is smaller; i.e. the volume fraction of particle is important. This change of phase is reversible and has a characteristic time about the millisecond.

Those properties and the efficient suspensions recently designed (good lifetime and temperature holding) open perspectives for "smart devices" applications [8,9]. However, these works are often delayed because of the lack of models characterizing efficiently the electrorheological fluids behavior.

In this paper, we are more specifically interested in the continuous model for the electromechanical behavior of electrorheological fluids in solid state subjected to oscillatory squeeze loads below the chains rupture yield. The oscillatory

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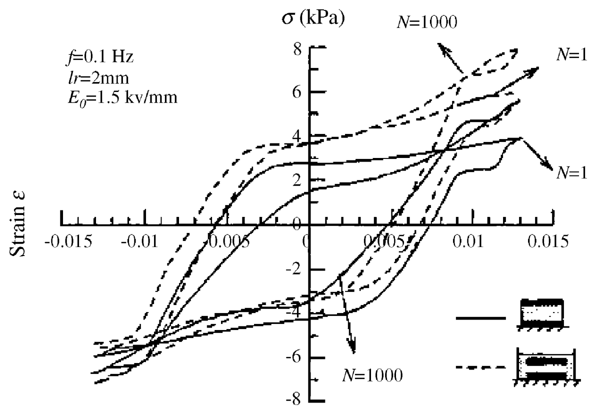

Fig. 1. Hysteresis loop of stress as a function of strain for a solid electrorheological fluid under an initial electric field $E^0 = 1.5\,\mathrm{kV/mm}$ [1].

squeeze tests carried out by Vieira et al. [1] seems to be the more significant. Fig. 1 presents the evolution of the stress $\sigma = F/S$ as a function of the strain $\varepsilon = \Delta h/h_0$ after application of an initial electric field and for small displacements driven by a triangular oscillatory signal. This test highlights a viscoelastic behavior of the material in solid state. Nevertheless, we can deplore that the authors did not measured the influence of a small variation of the electric field on the loop.

Continuous models for those materials have already been studied. We can quote works from Shkel and Klingenberg [10], however, they do not take into account dissipative phenomena experimentally observed. More recently, Napoli [5,13] proposed a modelling of dissipative behavior introducing an internal variable expressing the unit elongation of fibres.

In this context, our starting point was the description of continua whom electromechanical properties were developed, among others, by Eringen and Maugin [2,3]. This work has been limited to small mechanical and electric perturbations of a dielectric solid free of mobile electric charges and magnetic phenomena. Similarly to works done for small thermal and mechanical perturbations in thermoelasticity, we suppose that the mechanical behavior is not coupled with the electric one. The dissipative phenomena which are experimentally observed, are modeled by an internal variable which represents the chains microscopic deformation. We present a finite element resolution to our problem, based on a variational formulation using displacement, electric potential and the internal variable. We identify our model from the inverse analysis of Vieira's tests simulation. Next, we simulate the influence of a small perturbation of the electric field on the mechanical response of the material.

2. A continuum model

The modelling of electromagnetic and mechanical coupled phenomena require to couple Maxwell's equations, balance laws of mechanics and the thermodynamic principles. This coupling was developed in a general approach in literature (e.g. [2,3]) and allowed, for one, to model electromagnetostriction, piezoelectricity, ferromagnetism phenomena. Our work deals with a simplified version of equations stemming from the complete theory. We apply indeed no external magnetic field, and our medium is not magnetizable. As we neglect the presence of mobile charges, we set our model under the quasi-static approximation for Maxwell's equations. The response time, i.e. the time for the phase to change, is very short (about the millisecond), so we neglect the polarization inertia effects. The state variable chosen to describe our continuum are strain, temperature and electric field.

The initial configuration κ^0 (Fig. 2) is related to the material state when subjected to an initial electric field $\underline{E}^0$ before deformation. Particles are free to migrate in the suspending fluid, we consequently suppose that the application of an initial electric field do not create prestressed domains in the material. We assume the current configuration κ, to be obtained from a small mechanical perturbation (small displacements, small deformations) and a small electric perturbation (small perturbation of the initial electric field $\underline{E} = \underline{E}^0 + \delta\underline{E}$ with $\|\delta\underline{E}\| \ll \|\underline{E}^0\|$).

2.1. Governing equations

Let us consider a regular physical domain Ω occupied by an electrorheological fluid in solid state. The boundary of that domain is $\partial\Omega$ with its unit outward normal $\underline{n}$. The fields necessary to the description of media are

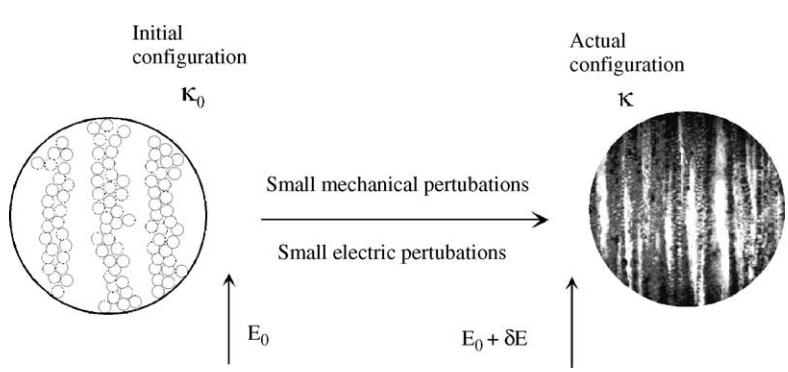

Fig. 2. Definition of initial configuration κ^0 and current configuration κ.