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Consistometers rheometry of power-law viscous fluids

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Abstract

This work provides a one-dimensional analytical theory for straight and axisymmetric consistometers, and helps with their use as rheometers. It is shown that these very economical and easy-to-use instruments provide an accurate characterisation of the rheological behaviour of shear-thinning, shear thickening and other materials of high apparent viscosity. The study may also be used heuristically for other types of flow, such as geological flows. The theoretical problem considered here may be presented as that of the transient gravity flow resulting from the total, instantaneous failure of a straight or curved dam on a horizontal base. A succession of analytical solutions is found, depending on the time that has elapsed since the moment the flow was released. One of these, relating to the viscous slump of the entire fluid, is particularly pertinent for deducing measurements of the wave front progress or slump, the flow curve and the rheological parameter values being involved in the mechanical behaviour of the material in question. Experiments conducted with a Newtonian glucose solution confirm the feasibility of rheometric characterisations and give rise to discussions on the range of validity of the theoretical predictions.

Keywords: Consistometer; Flow curve; Viscosity; Slump; Dam-break

1. Introduction

Laboratory rheometers are generally considered as a reliable and accurate means of measurement. In theory, they may be used not only to compare different materials from the point of view of their resistance to flow, but also to obtain the values of the fundamental physical parameters of the constitutive equations. However, the advanced technologies used in these rheometers make them expensive to buy. In addition, to obtain really accurate results, they call for a high level of expertise and require considerable experimental time. Commercially available rheometers are also fitted with fairly small measurement cells, and because of this they are unsuited to heterogeneous mixtures of millimetre-sized objects. In this respect, it is worth recalling the empirical rule given in [1], which must be followed in order to obtain the macroscopic properties of the materials. This involves using gaps that are systematically more than 50 times the length scale of the mesoscopic structure of the material.

For all these reasons at least, other means of measurement have been used for a long time in many sectors of industry. In the food industry, for instance, consistometers [2] such as Bostwick's consistometer are used. This is simply a straight, horizontal flume, part of which is closed by a movable gate forming a retaining "dam". The other part is initially empty. When the gate is suddenly opened, the volume of fluid initially at rest is caused to flow as a result of gravity. A wave is thus produced. The abscissa reached by the wave front at a given moment or the observed slump, can be used to compare the various substances being studied. The Adams consistometer operates in a similar manner, but is axisymmetric. These instruments are economical and robust, easy to use and clean.

The aim of this study is to establish analytical theoretical solutions for the scaling laws relating to changes in the wave front abscissa and slump as a function of time in the consistometers. The most pertinent formulae will be sought for evaluating with sufficient precision the values of the rheological parameters of a viscous fluid described by the power law. In an isochoric situation, the viscous behaviour considered may be represented as

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follows:

$$\boldsymbol{T} = 2\eta\left(\sqrt{-4D_{\mathrm{II}}}\right)\boldsymbol{D} \text{ and } \mathrm{tr}(\boldsymbol{D}) = 0 \quad (1)$$

$$\eta\left(\sqrt{-4D_{\mathrm{II}}}\right) = k\left(\sqrt{-4D_{\mathrm{II}}}\right)^{n-1} \quad (2)$$

where $\boldsymbol{T}$ and $\boldsymbol{D}$ are, respectively, the deviatoric stress tensor and strain rate tensor. The two scalars $D_{\mathrm{II}} = -(1/2)\mathrm{tr}(\boldsymbol{D}^2)$ and $\eta\left(\sqrt{-4D_{\mathrm{II}}}\right)$ are, respectively, the second invariant of $\boldsymbol{D}$ and the apparent viscosity. The latter is defined by two rheological parameters: the fluid consistency k and the power-law index n. Shear-thinning or shear-thickening behaviour is obtained when n is smaller or larger than 1, respectively. The case $n = 0$ is singular. It is legitimate to consider both isochoric and isothermal changes.

The fluid-experiment pairs concerned by this model are extremely varied [3]. Substances as diverse as food products and cosmetics, petroleum products, civil engineering and development materials and, coatings and inks, and others may be concerned, particularly when they are subjected to fairly high shear rates. At low shear rates, the effects of other rheological parameters may also be expected, such as those of yield stress [1,4], which will be discussed in a later article.

The average flow models used are an application of the more general one-dimensional theory for flows in long domains [5]. To begin with, they will be presented in Section 2. All developments up to Section 5 included will be devoted to the case of a straight Bostwick-type consistometer. The conservation equations lead to an inertia-dominated hyperbolic model that is pertinent for the first instants of flow, and to a viscous force-dominated parabolic model that is valid only a while after the gate has been opened. It may naturally be expected that the parabolic model will be preferred with a view to determining the rheometric properties of the fluids.

Section 3 will be devoted to solutions of the hyperbolic model. The analytical solution [6], in which wall friction is completely neglected, will be recalled very briefly. This solution is indeed helpful in following and understanding the succession of phenomena from the moment the gate is opened. It will be supplemented by an extension to the laminar case of solutions [7–9] already obtained in order to take into account turbulent delaying friction at the wave front.

Section 4 is devoted to the parabolic viscous model. At intermediate time scales, where the upstream fluid has not yet slumped, this problem will be solved with an upstream condition on the water depth of the Dirichlet type, $h = H$. The solutions proposed for the later instants, when the level drops in the upstream part of the reservoir, will then correspond to an upstream condition of the Neumann type, $\mathrm{d}h/\mathrm{d}x = 0$.

Section 5 will compare the theory and experiments carried out and limits will be sought for applying the different formulae.

In Section 6, the theoretical and experimental study is extended to the axisymmetric case of Adams type consistometers.

The conclusions are given in Section 7.

2. Hydraulics models for unsteady 2D flow on a horizontal plane

This study concerns the classic hydraulics problem [10] of a dam-break on a dry bed in a straight channel with a horizontal floor. It is assumed that the fluid adheres perfectly to the wall and that the effect of surface tension is negligible. The originality and novelty of the model developed here are found in the formulation of the non-linear viscous terms for a laminar flow.

As indicated in Fig. 1, the orthonormal coordinates O_{xy} define the abscissa x marked positive in the direction of flow from the mobile gate, and the ordinate y marked positive in the upwards direction from the horizontal floor. The depth of water h depends on x and on the time t.

The upstream fluid is initially at rest at depth $h = H$, between abscissae $-L$ and 0, and the gate opens suddenly at instant $t = 0$. The fluid then spreads on the dry floor between the abscissa of the upstream wall $-L$ and that of the front x_{f}, where $h = 0$. The fluid slumps under gravity effects, beginning in the immediate area of the gate and continuing until the entire heap becomes a long and increasingly flat strip of fluid.

Two of the usual approximations in hydraulics will be made here. The shallow water approximation expresses the thinness of the sheet of liquid compared with the wave length (conventionally, diagrams such as that of Fig. 1 are highly distorted). The approximation whereby the wall friction of the various flows is written in local variables means that formulae for developed flows may be used. With these, it is possible to simplify the mass and momentum conservation equations into two equations with two variables (space x and time t), relating to the mean discharge velocity in the section U and the water depth in the section h.

Details of the calculations have already been given in [5], in particular in normalized form. However, preference is given in free-surface hydraulics to dimensional presentations, and for this reason, the same thing will usually be done below.

By adopting a momentum form coefficient of 1 instead of $2(1 + 2n)/(2 + 3n)$, to avoid complicating the presentation

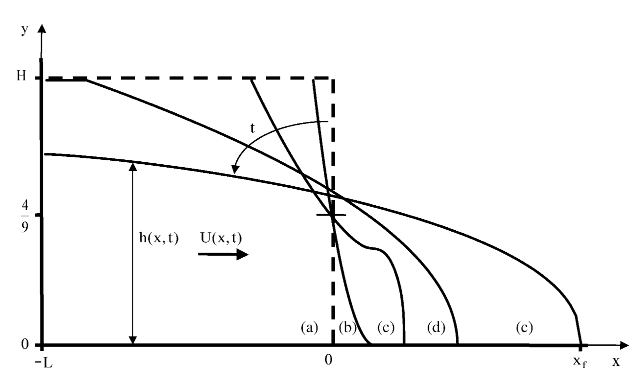

Fig. 1. Definition of notations and four successive free-surface flow regimes (b–e) after opening the gate at instant $t = 0$ (a).