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Welfare rankings from multivariate data, a nonparametric approach

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ABSTRACT

Economic and social welfare is inherently multidimensional. However, choosing a measure which combines several indicators is difficult and may have unintendend and undesirable effects on the incentives of policy makers. We develop a nonparametric empirical method for deriving welfare rankings for a social planner based on data envelopment, which avoids the need to specify a weighting scheme. We apply this method to data on Human Development.

1. Introduction

Sen and many others have consistently and persuasively argued that aspects of well-being, be they inequality, deprivation or polarization, are intrinsically many-dimensioned things (for example Sen (1995), Anand and Sen (1997), Atkinson (2003), Bourguignon and Chakravarty (2003), Kolm (1977), Maasoumi (1986) and the essays in Grusky and Kanbur (2006)). An individual's functionings and capabilities are bounded by many sensibilities, the extent of their freedoms, limitations afforded by their health, knowledge and skill set and ultimately their capacity to buy goods and leisure. Evaluation of these various aspects of societal wellbeing demands recognition of its multidimensional nature.

Whilst the argument that well-being is multidimensional is well taken it is often still extremely useful to be able to order and to compare states characterized in many dimensions. Policy makers, for example, frequently require some means of comparison that is complete. Thus beyond the difficulties surrounding measurement of these many sensibilities, an evaluation of overall well-being calls for some means of aggregating across them. Therein lies the difficulty, for while there may be general agreement on an aggregation method, the specific weights to be attached to each sensibility are a matter of some dispute. The choice of any particular weighting scheme is somewhat arbitrary, and unfortunately once made it rules out other equally plausible but no less arbitrary weighting schemes.

A good example of this problem is the United Nations Human Development Index (HDI) which aims to provide a single summary measure of the relative development status of different countries. Based upon indices of three dimensions, education (a combination of literacy and school enrolment rates), life expectancy and GDP per capita, it simply adds the three indices up and divides by three, attaching equal weight to each sensibility. The implication is a one percent increase in any one of the factors will have an effect on 'development' identical to that of a corresponding change in any other, and this will be the case whatever the levels of the individual factors. This has obvious implications for policy design, since a policy maker's attention will be directed to those factors which have the greatest weight in the aggregation scheme. Whether or not this is desirable should be a matter of conscious and careful consideration, rather than as the unintended consequence of the choice of a mathematical function.

This paper offers a constructive approach to the aggregation problem. We consider the situation in which we have data recording various aspects of well-being for a cross section of observations (life-expectancy, income and education, for example, for a cross section of countries as is the case for the UN HDI data). We show how two-sided bounds can be placed on a social planner's welfare index for each observation using only the assumptions that well-being is non-decreasing and weakly quasi-concave with respect to these indicators. Our approach is applied directly to the data and is fully nonparametric in the sense that it does not require us to make any further assumptions on the functional form of the welfare function, nor does it require us to estimate any functions of the data. Indeed the method we are suggesting can be applied to very small datasets (as well as to large ones) where statistical techniques – and especially nonparametric statistical techniques – could not be relied upon. A useful feature of our approach is that, since it is nonparametric and nonstochastic, the methodology is easily replicable requiring nothing more complex than standard linear programming techniques. We illustrate the method using recent UN HDI data. We show that it is

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indeed possible to recover informative two-sided bounds on the welfare index. Because the bounds encompass the entire set of welfare indices consistent with monotonicity and quasi-concavity, these bounds can be used as a computationally convenient robustness check on parametric methods. In other words researchers do not have to go through the unending tasking of computing all of the alternative measures, but instead simply have to compute the bounds. The approach set out in this paper also suggests a potential research program which might extend the work described in a number of ways.

The plan of the paper is as follows. Section 2 sets out the basic theory relating to our approach, describes the calculation of the bounds and provides two key propositions concerning them. Section 3 provides an empirical illustration which uses the UN HDI data and describes our experience with applying the methodology. Section 4 concludes and considers the shape of future work in this area.

2. Theory

Suppose that there are m variables recording different aspects of social and economic welfare for each of n observations in a dataset (this dataset may be composed of individuals, communities or countries and is indexed $i=1,\ldots,n$). In what follows we assume either that these variables are non-negative, or are transformed to be such. Let $\mathbf{x}_i \in \mathbb{R}_+^m$ denote the i'th observation. Let $\mathbf{X}$ be the $(m \times n)$ matrix of all of the n observations.

Let $W: \mathbb{R}_+^m \to \mathbb{R}$ denote a function which aggregates the variables associated with an observation into a single scalar measure. We think of W as representing a welfare/well-being function of a paternalistic social planner so that $W(\mathbf{x}_i)$ measures the social planner's view of the welfare of i'th observation. Level sets of the function W indicate how the social planner trades off gains in one dimension for losses in another. The UN HDI is an example of such a function: in this particular case, the trade-off is independent of the levels of the individual variables, and education and GDP, for example, are viewed as perfect substitutes regardless of the level at which they are present.

We will make the following two assumptions regarding the welfare function:

A1. Monotonicity: $W(\mathbf{x}) \geq W(\mathbf{y})$ if $\mathbf{x} \geq \mathbf{y}$.

A2. Quasi-concavity: $W(\mathbf{x}) = W(\mathbf{y}) \leq W(\alpha\mathbf{x} + (1-\alpha)\mathbf{y}) \forall \alpha \in [0,1]$.

Monotonicity means that the well-being does not fall with an increase in the measured variables. Quasi-concavity means that for a given distribution of $\mathbf{x}$, welfare is (weakly) increased by any inequality reducing reallocation between observations.

2.1. The distance function

In this paper we focus, not on the primal welfare function, but on a dual representation of it called the *distance function.*[1] The distance function measures the amount by which one has to scale the variable vector of an observation so that it achieves some reference welfare level. It is defined as follows:

$$d(\mathbf{x}, W) = \min_{d \geq 0} \{d : W(d\mathbf{x}) \geq W\} \tag{1}$$

The distance function is decreasing in $\mathbf{x}$, increasing in W and homogeneous of degree one in $\mathbf{x}$. The distance index can thought of as a (Malmquist) quantity index number measuring the '*size*' of $\mathbf{x}$ relative to the reference welfare level W.[2] To illustrate consider Fig. 1

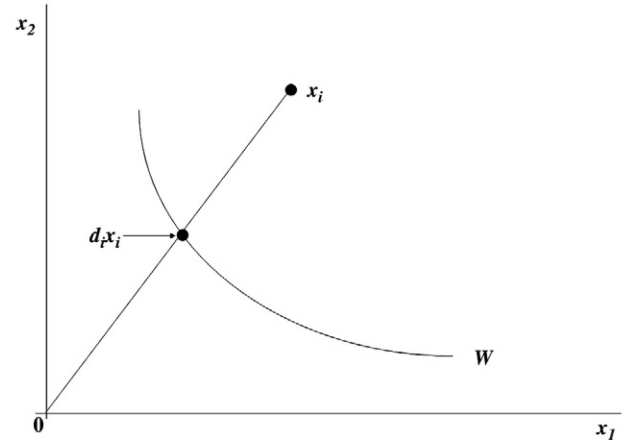

Fig. 1. The distance function.

which shows the general idea behind this index. There are two variables $\{x_1, x_2\}$, one measured on each axis and a single observation $(\mathbf{x}_i)$. The curve W represents all of the combinations of the two variables which can produce a reference level of welfare. This curve is downward-sloping and convex to the origin thanks to the two assumptions above. The value of the distance function is given by the scalar value d_i. This is the smallest number by which $\mathbf{x}_i$ can be scaled such that the bundle $d_i\mathbf{x}_i$ lies on or above W. In this case $d_i \approx \frac{1}{2}$ which means that an equi-proportional reduction of about 50% in all of the variables would place the observation at the required reference welfare. Lower (respectively higher) values of d_i indicate higher (lower) welfare compared to W. That distance functions in general depend on the location of $\mathbf{x}_i$ the welfare function and the reference welfare level is clearly illustrated by the figure by considering how the construction would vary with these factors. Another feature which is implicit in the figure is that knowing the distance function is as good as knowing the welfare function itself (you can identify the curve by knowing the value of d_i for all possible locations of $\mathbf{x}_i$ and connecting up the set of points such that $d_i = 1$).

Since the distance function is a dual representation of the welfare function we could choose a formula for either and proceed to apply them a dataset in order to investigate welfare rankings. However, given the forgoing discussion about the difficulties involved in agreeing on a specific welfare aggregator, the challenge is to try to develop methods which are nonparametric; that is, which do not depend upon the functional form of a specific aggregator. In the next section we show that it is possible to recover bounds on the distance function which are valid for all possible choices of aggregator which satisfy monotonicity and quasi-concavity given an appropriate choice of the reference observation.

2.2. Bounding the distance function

Consider the following reference welfare level:

$$W^* = \min_j \left\{ W\left(\mathbf{x}_j\right) : \mathbf{x}_j \in \mathbf{X}, W \text{ satisfies A1 and A2} \right\}$$

That is, the reference welfare level is the welfare associated the worst-off observation where the welfare measure is required to satisfy monotonicity and quasi-concavity. Given this reference welfare curve it is possible to recover two-sided bounds on the distance index for each observation in the data without making further parametric assumptions about the welfare function. The formal result is stated next.

[1] See, for example, Deaton (1979) and Deaton and Muellbauer (1980). The term is from the economics literature (Shephard (1953) for example). In the mathematics literature the same object is known as a gauge function (see Rockafellar (1970), for example).

[2] This is a standard method in the index number literature. See Malmquist (1953).