# Flutter Control of a Two-dimensional Airfoil Using Wash-out Filter Technique

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**Abstract:** The wash-out filter (WF) technique is used to control the flutter of a two dimensional airfoil with cubic non-linearity in incompressible flow. Firstly, Hopf bifurcation theory is used to determine the point at which the nonlinear controller is introduced. The system is then transformed into Jordan canonical form, based on analysis of linearized eigenvalues of the system. Secondly, for the introduced WF controller, the linear control gain is determined according to Hopf bifurcation condition. The symbolic computing program of normal form direct method (NFDM) is also used to obtain the normal form of the controlled system. The non-linear control gain can be determined based on the relation of the type of bifurcation and the parameters of the normal form, to transform sub-critical Hopf bifurcation to be super-critical one. Lastly, numerical simulations are used to certify the validity of theoretical analysis, in which the amplitude of flutter or limit cycle of the controlled system is reduced greatly, comparing to the original system.

**Key words:** nonlinear airfoil; flutter control; wash-out filter (WF); Hopf bifurcation; normal form direct method (NFDM)

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摘　要: 研究应用 wash-out 滤波器技术对具有立方非线性俯仰刚度的二元机翼颤振的控制。首先,确定需要引入 Hopf 分岔的点,并在该点将原系统方程 Jordan 化;其次,对于引入的 wash-out 滤波控制器,先按 Hopf 分岔条件确定线性控制增益,再用规范型直接法得到受控系统的规范型,由分岔类型与规范型系数的关系确定非线性控制增益,从而将原系统的亚临界 Hopf 分岔变为超临界 Hopf 分岔;最后通过数值模拟验证了控制的有效性,并发现受控系统的颤振幅值(极限环大小)大大降低。

关键词: 非线性机翼; 颤振控制; wash-out 滤波器技术; Hopf 分岔; 规范型直接法

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For a flight airfoil, flutter is a dynamic aeroelastic instability that involves structure elastic, inertial and aerodynamic forces. Generally, flutter occurs when a critical flutter speed is exceeded. If flutter occurs in flight, the aircraft structure may fail. Therefore, it is important to predict the aeroelastic characteristics accurately to prevent the occurrence of flutter.

During the past decades, many aeroelastic analyses of flight vehicles have been performed. Typically, nonlinear aeroelastic responses include flutter, divergence, limit cycle oscillation (LCO) and chaotic motion. Zhao and Yang[1, 2] analyzed the LCO, period doubling motion and chaotic motion using the numerical integration and the harmonic balance method respectively. Liu[3] studied the typical bifurcation point using the successor function method. Price *et al.*[4] and Singh *et al.*[5], respectively, investigated the flutter characteristics in the time and the frequency domains by the describing function method. Ding *et al.*[6] improved the cell mapping method and applied it to global analysis of aeroelastic system with bilinear structural stiffness. Different types of motions including damped stable motion, LCO, complicated periodic motion, chaotic motion and divergent flut-

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ter were determined as a function of initial conditions (domains of attraction). Ding and Wang[7] studied the influence of structural coefficients on the topological structure of the Hopf bifurcation of the airfoil flutter. They found that depending on value of the linear stiffness coefficient, the Hopf bifurcation can be subcritical, a catastrophic type of flutter, and super-critical, a benign type of flutter. Obviously the sub-critical bifurcation should be avoided.

Bifurcation phenomenon is very complex. It can bring the dynamical system to different extents of danger and even disaster. Therefore, it is necessary to control the bifurcation phenomenon in real-time manner, to change the bifurcation from catastrophic type to benign one. Introducing intently the bifurcation controller can modify the dynamical characteristics of system, which includes postponing the occurrence of Hopf bifurcation, stabling the unstable bifurcation orbit, modifying the type or form of bifurcation orbit and controlling the chaotic motion through bifurcation control, and so on. The bifurcation controls have been widely used in such as biological medical engineering, aeronautic and aerospace engineering, and electricity system. The common used methods include the wash-out filter (WF) controller, linear and nonlinear feedback methods, frequency domain analysis, approaching approximation and normal form theory based method.

The WF method is expanded from linear or nonlinear feedback method. Utilizing the principle of bifurcation anti-control, the WF method can modify the system dynamics through introducing a new bifurcation. It has following advantages: simple controller structure; easy engineering implementation; small control cost in controlling bifurcation or chaotic; and being the same with multi-dimensional system and with certain robust. So the WF technique has been widely used in the engineering field. These include controlling the chaotic phenomenon of Lorenz system[8]; improving the complex dynamics of bifurcation and chaos of heart with unusually pulses, through adjusting the heart pulse and as well the alternative impulsion of the two heart chambers[9]; and controlling the chaos of the system under parametric disturbance[10].

In this paper, the active control on flutter of a two-dimensional airfoil system with structural cubic non-linearity is investigated by using the WF technique and the normal form directed method (NFDM). Firstly, the linear gain of the nonlinear controller is determined by using WF technique. Then the normal form of the controlled system is calculated by using the NFDM, to illustrate the relation of the topological bifurcation structure of the controlled system and the nonlinear gain of the nonlinear controller. With proper value of nonlinear gain, the sub-critical Hopf bifurcation can be suppressed by introducing a super-critical Hopf bifurcation intently, to improve the stability of flight airfoil.

## 1 Bifurcation of Airfoil with Cubic Non-linearity

For airfoil flying in incompressible flow, with viscous damp and cubic nonlinear pitch stiffness, the equations of motion of the two-degree-of-freedom aeroelastic system are described as[1]

$$\left.\begin{aligned} \ddot{h} + 0.25\ddot{\alpha} + 0.1\dot{h} + 0.2h + 0.1Q\alpha &= 0 \\ 0.25\ddot{h} + 0.5\ddot{\alpha} + 0.1\dot{\alpha} + k\alpha + e\alpha^3 - 0.04Q\alpha &= 0 \end{aligned}\right\} \tag{1}$$

where $h$ and $\alpha$ are the plunging displacement and the twist angle about the pitch axis. $k$, $e(=20)$ and $Q$ are the linear pitch stiffness coefficient, nonlinear stiffness factor and the air speed, respectively. Flutter analysis[7, 11] showed that there exists a critical linear stiffness coefficient, $k_0 \approx 0.126$, that the case $k > k_0$ results a super-critical bifurcation and the case $k < k_0$ results a sub-critical bifurcation. Fig. 1 shows the global bifurcation diagram for sub-critical Hopf bifurcation ($k = 0.0816$). The linear bifurcation point, obtained from the stability analysis, is $B_1 = 1.5$. Nevertheless, the trivial solution of the system becomes local stable as the air speed $Q > Q_C \approx 0.9$, which means that a large enough disturbance or initial condition