# Nonlinear Dynamic Analysis of Planar Flexible Underactuated Manipulators

HE Guang-ping[1], LU Zhen[2]

(1. School of Mechanical and Electrical Engineering, North China University of Technology, Beijing 100041, China)

(2. School of Automation Science and Electrical Engineering, Beijing University of Aeronautics and Astronautics, Beijing 100083, China)

**Abstract:** The influence of passive joint on the dynamics of planar flexible underactuated manipulators is studied. A vibration reduction method based on the internal resonance phenomenon of multi-degree nonlinear dynamic system is proposed. The dynamic simulation results reveal that the harmonic input for the actuated joint will induce the passive joint deviating from its equilibrium position, the excursion speed and direction depend on the amplitude of the input. A passive joint position control scheme making use of the vibration of the flexible structure is suggested, and the numerical simulation results of a model of planar two-link flexible underactuated manipulator is shown.

**Key words:** dynamics and vibration; manipulator; underactuated; nonlinear; internal resonance

平面柔性欠驱动机械臂的非线性动力学分析. 何广平,陆 震. 中国航空学报(英文版), 2005, 18(1): 78-82.

摘 要:研究了平面柔性欠驱动机械臂中被动关节对系统动态特性的影响,在动力学分析的基础上,提出了一种基于内共振原理的柔性欠驱动机械臂振动控制方法。在对系统的稳态周期运动进行分析研究的基础上,发现处于自由摆动状态的被动关节的平衡位置随系统结构的振动而发生漂移,并且被动关节平衡位置的漂移速度和方向与周期输入的振幅有关。提出利用结构柔性产生的振动实现被动关节位置控制的方法,通过平面二连杆柔性欠驱动机械臂进行了仿真计算。

关键词:动力学与振动; 机械臂; 欠驱动; 非线性; 内共振

文章编号: 1000-9361(2005)01-0078-05 中图分类号: TP242 文献标识码: A

The motion control of underactuated mechanism or underactuated manipulator is proposed in the fault tolerance technology of space robot system firstly[1]. An underactuated manipulator also can be designed as a kind of assistant robot system, which is said to be a COBOT[2]. The COBOT system can not work independently for the passive joint is unable to provide the force/torque for manipulation under gravity. However, the COBOT can cooperate with human being and implement some critical manipulation such as applications in bioengineering, medical treatment and micro-electronics. A metamorphic mechanism[3] can reconfigure itself, and always results in the change of degree of freedoms (DOFs) or constraint characteristics of the original system. Metamorphic mechanisms have some passive joints or flexible components for economical sake and make the best of the ability of reconfiguration of the mechanism. Therefore, the underactuated mechanisms and underactuated manipulators have some properties which can not be provided by the full-actuated mechanism.

Most of the studies on the underactuated manipulators assumed that the structure was rigid and the research was limited in position control[4,5] of passive joint or motion plan of system mainly. However, the underactuated manipulator always is flexible in structure for its application background obviously. A machine or manipulator is designed so

Received date: 2004-02-26; Revision received date: 2004-07-05
Foundation item: National Natural Science Foundation of China (50375007, 50475177)

rigid or strong that it has enough accuracy in manipulation generally. Nevertheless, for the sake of economy of material or energy, the dynamic behavior of the actual manipulator considering the structural flexibility is critical and has to be investigated thoroughly. On the other hand, the flexible manipulator has some merits such as the characteristic in structural compliancy or economy in energy consuming. Therefore, the flexible system is another important developing direction of the robotics currently[6].

In this paper, the nonlinear vibration behaviors of the flexible underactuated manipulator are investigated based on the internal resonance property of multi-degree nonlinear dynamic system, and a vibration reduction method is proposed. Moreover, the influence of the structural flexibility on the motion behavior of the passive joint is explored, and a position control method for passive joint through the structural vibration is suggested.

## 1 Dynamic Formulation

Considering an $n$-DOFs open chain planar flexible manipulator, the vibration of the system is induced from the transverse bending of the link and the elasticity of the joint. The passive joints in the underactuated manipulator have no elastic components, so the stiffness of the passive joint is zero. Assume that the planar manipulator is horizontal (Fig. 1) and the passive joints have brakes, which

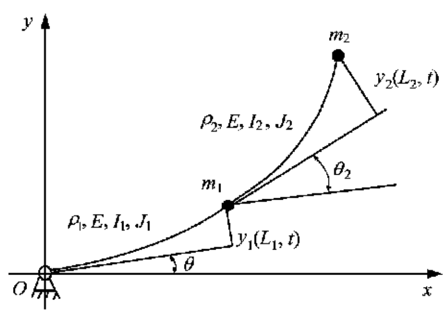


Fig. 1 Planar two-link flexible underactuated manipulator model

control the passive joint to freely swing or to be locked. The length of link $i$ is $L_i$, the moment of inertia of the cross-section area is $I_i$, the materials of the links are same, $E$ is Yong's modulus, the mass of unit length of the link is $\rho_i$, and $m_i$ ($i = 1, 2, \cdots, n$) is the payload of each link end. Assume that the link is a Euler-Bernouli beam.

The deformation of link described by the assumed mode method can be written as

$$y_i(x, t) = \sum_{j=1}^{m} \phi_{(i-1)j}(x)\,\delta_{(i-1)j}(t) + \sum_{j=1}^{m} \phi_{ij}(x)\,\delta_{ij}(t) \quad (i = 1, 2, \cdots, n) \qquad (1)$$

where $\phi_{ij}(x)$ is the $j$-th assumed mode function of link $i$, and $\phi_{0j}(x) = 0$, $\delta_{ij}(t)$ are the mode amplitudes. The mode shape function

$$\phi_{ij}(x) = \mathrm{sh}(\beta_{ij}x) + \xi_{ij}\sin(\beta_{ij}x) \qquad (2)$$

satisfies the boundary conditions for a simple supported beam with one end free. In Eq. (2), the parameter

$$\xi_{ij} = \mathrm{sh}(\beta_{ij}L_i)/\sin(\beta_{ij}L_i) \quad (j = 1, 2) \qquad (3)$$

and

$$\beta_{ij}L_i \approx \left(j + \frac{1}{4}\right)\pi \quad (j = 1, 2) \qquad (4)$$

By Lagrange equation,

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\delta}_i} - \frac{\partial L}{\partial \delta_i} = \tau_i \quad (i = 1, 2, \cdots, n) \qquad (5)$$

The dynamic equation in matrix-vector form is given by

$$\boldsymbol{M}\ddot{\theta} + \boldsymbol{K}\theta + \boldsymbol{c} = \tau \qquad (6)$$

where, $\theta \in \mathbf{R}^{(n+nm)\times 1}$ is the generalized coordinate; $\boldsymbol{M} = [m_{ij}] \in \mathbf{R}^{(n+nm)\times(n+nm)}$ is the inertia matrix; $\boldsymbol{K} = [k_{ij}] \in \mathbf{R}^{(n+nm)\times(n+nm)}$ is the stiffness matrix; $\boldsymbol{c} \in \mathbf{R}^{(n+nm)\times 1}$ contains the Coriolis, centrifugal, elastic and centrifugal stiffening effects; $\tau \in \mathbf{R}^{(n+nm)\times 1}$ denotes the generalized external torque.

## 2 Dynamic Analysis

### 2.1 The vibration reduction effect with passive joints

The multi-degree nonlinear dynamic system such as Eq. (6) can be analyzed by many approximation methods such as averaging method, progression-approximating technique, harmonic equilibrium and so on. It has been shown that the system has a plenty of dynamic phenomena. Howev-