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Numerical comparison of the method of transport to a standard scheme

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Abstract

In previous joint work with Zimmermann, we derived Fey's method of transport (MoT), a multidimensional flux vector splitting scheme, from gas kinetic theory via quadrature. Now we present a number of numerical tests in one and two space dimensions showing that similarly to many other flux vector splitting and kinetic schemes, the MoT is very dissipative. In order to quantify its numerical dissipation, we compare the second order MoT-ICE [J. Comput. Phys. 164(2) (2000) 283] to the classical, very simple, and at the same time very robust Harten–Lax–van Leer (HLL) scheme (also in a second order version), whose most significant drawback is its comparatively high dissipativity. Our numerical experiments indicate that the MoT-ICE is approximately as dissipative as the much cheaper HLL scheme. Recall that the HLL scheme does not suffer from any of the known multidimensional instabilities, which have been a prime motivation for the development of Riemann solver free schemes.

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1. Introduction

Many physical problems (for instance gas dynamics, magnetohydrodynamics, traffic flow) can be modeled by systems of hyperbolic conservation laws,

$$\partial_t \boldsymbol{U} + \nabla_{\underline{x}} \cdot \underline{\boldsymbol{F}}(\boldsymbol{U}) = 0, \tag{1.1}$$

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if certain effects (for instance viscosity) are neglected. One of the first algorithms to compute solutions to conservation laws was introduced by Godunov [7] in 1959. His scheme is based on exactly solving a Riemann problem at each cell interface and then projecting the solution back onto the space of piecewise constant functions. An efficient variant of Godunov's method are the simplified 'approximate' Riemann solvers introduced by Roe [15], Harten et al. [8], Osher and coworkers [3,12], and others.

Due to the enormous complexity of the multidimensional Riemann problem, Godunov's approach cannot be applied directly to multidimensional systems. However, there are some standard techniques to extend his scheme to more space dimensions, e.g. dimensional splitting (on a Cartesian mesh) or a finite volume framework (on an unstructured mesh). Piecewise polynomial reconstructions and a Runge–Kutta time step can be used to increase the formal order of accuracy. For a detailed description of Riemann solvers and Riemann solver based schemes see the textbook of Toro [21].

Despite the great success of these schemes, there is an ongoing discussion—started by Roe himself in 1986 [16]—whether one-dimensional Riemann solvers do justice to the multidimensional effects arising in such systems. In 1994, Quirk [14] presented a number of typical examples where many of the traditional Godunov-type schemes fail, unless they are stabilized by adding numerical viscosity. Riemann solver free schemes claim to handle multidimensional effects better, so that these problems are avoided.

The present article focuses on one of these Riemann solver free schemes, Fey's method of transport (briefly called MoT), derived in [4–6]. In previous work of the authors jointly with Zimmermann [10], we presented some new connections between several Riemann solver free schemes. One result was a new derivation, from gas kinetic theory, of those state and flux decompositions on which Fey's method is based. Now, we numerically compare the MoT in its ICE version [11] to the Riemann solver of Harten, Lax, and van Leer [8] (briefly called HLL). The HLL scheme is known to be very simple and at the same time very robust but suffers from being rather dissipative: contact discontinuities or, more generally, density variations are smeared out considerably. On the other hand, Quirk [14] already pointed out that the damping effect of the HLL scheme prevents it from producing multidimensional instabilities or unphysical solutions. Moreover, Pandolfi and D'Ambrosio [13] analyzed two of the instabilities Quirk had observed, namely the carbuncle phenomenon and the odd–even-decoupling (the latter also known as cross flow-instability). They came to the conclusion that a scheme which damps out density variations usually produces correct results for this type of test problems. In fact, they classify the HLL scheme as a 'carbuncle-free scheme'. Conversely, schemes which keep contacts sharp can be expected to suffer from multidimensional instabilities. This is, for example, the case for the HLLC scheme, a modification of HLL which preserves stationary contact discontinuities (see [22]). Pandolfi and D'Ambrosio classify the HLLC scheme as 'strongly carbuncle prone'.

Carrying this discussion one step further, we would like to propose that a multidimensional scheme that claims to be better than Riemann solver based schemes should firstly be less dissipative than HLL, and at the same time it should be carbuncle-free. The numerical results presented below, however, show that the MoT does not meet the first requirement. In fact, it is at least as dissipative as HLL.

The diffusivity of the MoT-ICE did not become apparent in [11] because there Noelle only treated the shallow water equations, which do not contain the linear field present in the Euler